MATHEMATICS SAMPLE PAPER #05

ANSWER AND SOLUTIONS

SECTION-A

1. Option (4)
More than 3

2. Option (3)
$$\cos\theta = \frac{\sqrt{b^2 - a^2}}{b}$$

3. Option (2)
$a_n = 3.5$

4. Option (3)
Trigonometric ratios of the angles.

5. Option (3)
140°

6. Option (3)
10

7. Option (2)
2.1

8. Option (2)
$$\frac{5}{2}$$

9. Option (1)
360 cm^2

10. Option (4)
Median

11. Option (3)
2 and –2

12. Option (4)
4

13. Option (1)
0 or 4

14. Option (4)
7000

15. Option (2)
$\sqrt{34}$

16. Option (4)
0°

17. Option (2)
28

18. Option (2)
$$\frac{BE}{EC}$$

19. Option (4)
Assertion (A) is false but Reason (R) is true.

20. Option (2)
Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

SECTION-B

21. In ΔPAO and ΔQBO,
$\angle A = \angle B = 90°$ (Given)
$\angle POA = \angle QOB$ (Vertically opposite angles)
Since, $\Delta PAO \sim \Delta QBO$, (by AA similarity)

Then, $\frac{OA}{OB} = \frac{PA}{QB}$

or, $\frac{6}{4.5} = \frac{4}{QB}$

or, $QB = \frac{4 \times 4.5}{6}$

$\therefore$ QB = 3 cm

22. Here, the total number of possible outcomes = 5.
(i) Since, there is only one queen
$\therefore$ Favourable number of elementary events = 1

$\therefore$ Probability of getting the card of queen $= \frac{1}{5}$.

(ii) Now, the total number of possible outcomes = 4.
Since, there is only one ace
$\therefore$ Favourable number of elementary events = 1

$\therefore$ Probability of getting an ace card $= \frac{1}{4}$.

23. HCF × LCM = Product of two numbers

$9 \times 360 = 45 \times$ 2nd number

2nd number = 72

OR

Let us assume, to the contrary that $7-\sqrt{5}$ is rational

$7-\sqrt{5} = \frac{p}{q}$, where p & q are co-prime and $q \neq 0$

$\Rightarrow \sqrt{5} = \frac{7q-p}{q}$

$\frac{7q-p}{q}$ is rational = $\sqrt{5}$ is rational which is a contradiction

Hence $7-\sqrt{5}$ is irrational

24. 20^{th} term from the end = $\ell - (n-1)d$

$= 253 - 19 \times 5$

$= 158$

OR

$7a_7 = 11a_{11}$

$\Rightarrow 7(a + 6d) = 11(a + 10d)$

$\Rightarrow 4a + 68d = 0$

$\Rightarrow a + 17d = 0$

$\Rightarrow a_{18} = 0$

25. $x = \frac{6-6}{5} = 0$

$y = \frac{-10+15}{5} = 1$

Hence, coordinates of point P(0, 1)

SECTION-C

26. Let the numerator be x and denominator be y.

$\therefore$ Fraction = $\frac{x}{y}$

Now, according to question,

$\frac{x-1}{y} = \frac{1}{3} \quad \Rightarrow \quad 3x - 3 = y$

$\therefore 3x - y = 3$ ....(i)

and $\frac{x}{y+8} = \frac{1}{4} \quad \Rightarrow \quad 4x = y + 8$

$\therefore 4x - y = 8$ ....(ii)

Now, subtracting equation (ii) from (i), we have

$$\begin{array}{r} 3x - y = 3 \\ 4x - y = 8 \\ - \quad + \quad - \\ \hline -x = -5 \end{array}$$

$x = 5$

Putting the value of x in equation (i), we have

$3 \times 5 - y = 3 \Rightarrow 15 - y = 3 \Rightarrow 15 - 3 = y$

$\therefore y = 12$

Hence, the required fraction is $\frac{5}{12}$.

OR

Let the speed of car at A be x km/h

And the speed of car at B be y km/h

Case 1 $8x - 8y = 80$

$x - y = 10$

Case 2 $\frac{4}{3}x + \frac{4}{3}y = 80$

$x + y = 60$

On solving x = 35 and y = 25

Hence, speed of cars at A and B are 35 km/h and 25 km/h respectively

27. LHS = $\sin\theta(1 + \tan\theta) + \cos\theta(1 + \cot\theta)$

$= \sin\theta + \sin\theta \cdot \frac{\sin\theta}{\cos\theta} + \cos\theta + \cos\theta\frac{\cos\theta}{\sin\theta}$

$= (\sin\theta + \cos\theta) + \frac{\sin^2\theta}{\cos\theta} + \frac{\cos^2\theta}{\sin\theta}$

$= (\sin\theta + \cos\theta) + \frac{\sin^3\theta + \cos^3\theta}{\sin\theta\cos\theta}$

$= (\sin\theta + \cos\theta)\left[1 + \frac{\sin^2\theta + \cos^2\theta - \sin\theta\cos\theta}{\sin\theta\cos\theta}\right]$

$= (\sin\theta + \cos\theta)\left[1 + \dfrac{1}{\sin\theta\cos\theta} - 1\right]$

$= (\sin\theta + \cos\theta) \times \dfrac{1}{\sin\theta\cos\theta}$

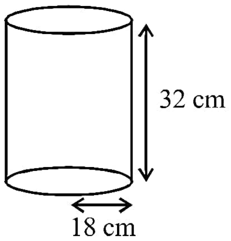
$= \dfrac{1}{\cos\theta} + \dfrac{1}{\sin\theta}$

$= \sec\theta + \text{cosec}\theta$

= RHS

Hence proved

28. Volume of cylindrical bucket = Volume of conical heap of sand.



$\pi r^2 h = \dfrac{1}{3}\pi R^2 \times 24$

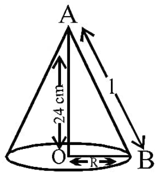
$\pi \times 18 \times 18 \times 32$

$= \dfrac{1}{3}\pi R^2 \times 24$

$R^2 = \dfrac{18 \times 18 \times 32 \times 3}{24} = \dfrac{18 \times 18 \times 32 \times 3}{24}$

R = 36 cm

In the ΔAOB of conical heap.



$AB^2 = AO^2 + OB^2$

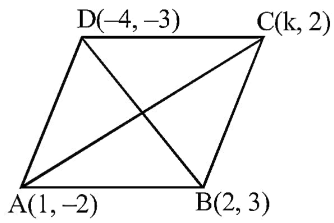
$\ell^2 = 24^2 + 36^2$

$\ell = \sqrt{576 + 1296}$

$= \sqrt{1872}$

ℓ = 43.27 cm = 43.3 cm

29.



Diagonals of parallelogram bisect each other

⇒ midpoint of AC = midpoint of BD

$\Rightarrow \left(\dfrac{1+k}{2}, \dfrac{-2+2}{2}\right) = \left(\dfrac{-4+2}{2}, \dfrac{-3+3}{2}\right)$

$\Rightarrow \dfrac{1+k}{2} = \dfrac{-2}{2}$

⇒ k = –3

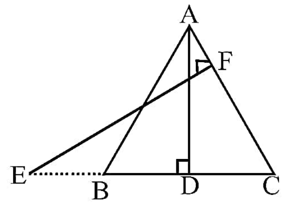
30. 200 – 250 is the modal class

$\text{Mode} = \ell + \dfrac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$

$= 200 + \dfrac{12-5}{24-5-2} \times 50$

= 200 + 20.59 = ₹220.59

31.



In ΔABD and ΔCEF

AB = AC (Given)

⇒ ∠ABC = ∠ACB

(Equal sides have equal oppposite angles)

∠ABD = ∠ECF

∠ADB = ∠EFC [Each 90°]

So, ΔABD ~ ΔECF (AA – Similarity)

OR

∠1 = ∠2 (Given)

PT = PS

(Side opposite to equal angles are equal) ....(1)

ΔNSQ ≅ ∠MTR

∠Q = ∠R (by cpct)

PQ = PR

(Side opposite to equal angles are equal) ....(2)

From (1) and (2)

$\dfrac{PT}{PR} = \dfrac{PS}{PQ}$

∠P = ∠P

ΔPTS ~ ΔPRQ (by SAS Similarity)

SECTION-D

32. $S_n = \dfrac{n}{2}[2a + (n-1)d]$

$S_{30} = \dfrac{30}{2}[2a + 29d] \Rightarrow S_{30} = 30a + 435d$ ...(i)

$\Rightarrow S_{20} = \dfrac{20}{2}[2a + 19d] \Rightarrow S_{20} = 20a + 190d$

$S_{10} = \frac{10}{2}[2a + 9d] \Rightarrow S_{10} = 10a + 45d$

$3(S_{20} - S_{10}) = 3[20a + 190d - 10a - 45d]$

$= 3[10a + 145d] = 30a + 435d = S_{30}$

[From (i)]

Hence, $S_{30} = 3(S_{20} - S_{10})$ Hence proved.

OR

Sum of first seven terms,

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_7 = \frac{7}{2}[2a + (7-1)d] = \frac{7}{2}[2a + 6d]$

$\Rightarrow 63 = 7a + 21d$

$\Rightarrow a = \frac{63 - 21d}{7}$ ... (1)

$\Rightarrow S_{14} = \frac{14}{2}[2a + 13d]$

$\Rightarrow S_{14} = 7[2a + 13d] = 14a + 91d$

But ATQ,

$S_{1-7} + S_{8-14} = S_{14}$

$63 + 161 = 14a + 91d$

$\Rightarrow 224 = 14a + 91d$

$2a + 13d = 32$

$2\left(\frac{63 - 21d}{7}\right) + 13d = 32$ (from 1)

$\Rightarrow 126 - 42d + 91d = 224$

$\Rightarrow 49d = 98$

$\Rightarrow d = 2$

$\Rightarrow a = \frac{63 - 21 \times 2}{7} = \frac{63 - 42}{7} = 3$

$\Rightarrow a_{28} = a + 27d = 3 + 27 \times 2$

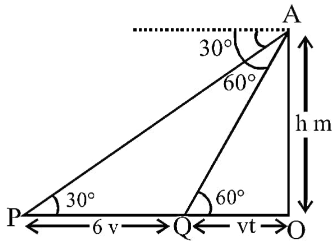
$\Rightarrow a_{28} = 3 + 54 = 57$

33. Let OA be the tower of height h, and P be the initial position of the car when the angle of depression is 30°.

After 6 seconds, the car reaches to Q such that the angle of depression at Q is 60°. Let the speed of the car be v metre per second. Then,

PQ = 6v (∵ Distance = speed × time)

and let the car take t seconds to reach the tower OA from Q (Figure). Then OQ = vt metres.



Now, in ΔAQO we have

$\tan 60° = \frac{OA}{QO}$

$\Rightarrow \sqrt{3} = \frac{h}{vt} \Rightarrow h = \sqrt{3}\ vt$ ....(i)

Now, in ΔAPO, we have

$\tan 30° = \frac{OA}{PO}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{6v + vt} \Rightarrow \sqrt{3}h = 6v + vt$ ....(ii)

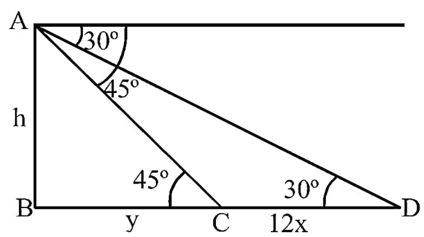
Now, substituting the value of h from (i) and into (ii), we have

$\sqrt{3} \times \sqrt{3}\ vt = 6v + vt$

$\Rightarrow 3vt = 6v + vt \Rightarrow 2vt = 6v \Rightarrow t = \frac{6v}{2v} = 3$

Hence, the car will reach the tower from Q in 3 seconds.

OR



Let the speed of car be x m/ minutes

In ΔABC

$\frac{h}{y}$ = tan 45°

⇒ h = y

In ΔABD

$\frac{h}{y+12x}$ = tan 30°

⇒ $h\sqrt{3}$ = y + 12x

⇒ $y\sqrt{3}$ − y = 12x

⇒ $y = \frac{12x}{\sqrt{3}-1} = \frac{12x(\sqrt{3}+1)}{2}$

⇒ y = 6x ($\sqrt{3}$ + 1)

Time taken from C to B = 6($\sqrt{3}$ + 1) minutes

34. In ΔAPE and ΔBPF,

∠APE = ∠BPF [Vertically opposite angles]

∠AEP = ∠BFP [Alternate angles]

By AA similarity, ΔAPE ~ ΔBPF

Thus, $\frac{AP}{BP} = \frac{PE}{PF} = \frac{AE}{BF}$ ....(1)

In ΔCPE and ΔDPF,

∠CPE = ∠DPF [Vertically opposite angles]

∠CEP = ∠DFP [Alternate angles]

By AA similarity, ΔCPE ~ ΔDPF

Thus, $\frac{CP}{DP} = \frac{PE}{PF} = \frac{CE}{DF}$ ....(2)

In ΔAPC and ΔBPD,

∠APC = ∠BPD [Vertically opposite angles]

∠ACP = ∠BDP [Alternate angles]

By AA similarity, ΔAPC ~ ΔBPD

Thus, $\frac{AP}{BP} = \frac{PC}{PD} = \frac{AC}{BD}$ ....(3)

From equations (1), (2) and (3), we get

$$\frac{AE}{BF} = \frac{AC}{BD} = \frac{CE}{FD}$$

Hence proved

35.

Class Interval	Frequency	cf
0 – 100	2	2
100 – 200	5	7
200 – 300	x	7 + x
300 – 400	12	19 + x
400 – 500	17	36 + x
500 – 600	20	56 + x
600 – 700	y	56 + x + y
700 – 800	9	65 + x + y
800 – 900	7	72 + x + y
900 – 1000	4	76 + x + y

N = 100

⇒ 76 + x + y = 100

⇒ x + y = 24 ...(i)

Median = 525

⇒ 500 – 600 is median class

$$\text{Median} = \ell + \frac{\frac{n}{2} - cf}{f} \times h$$

⇒ $500 + \left(\frac{50-36-x}{20}\right) \times 100 = 525$

⇒ (14 – x) × 5 = 25

⇒ x = 9

⇒ from (1), y = 15

SECTION-D

36. (i) Let the fixed charge for two days be Rs.x and additional charge be Rs.y per day.

As Radhika has taken book for 4 days.

It means that Radhika will pay fixed charge for first two days and pays additional charges for next two days.

x + 2y = 16

(ii) As the fixed charge for two days be Rs.x and additional charge be Rs.y per day

It means that Amruta will pay fixed charge for first two days and pays additional charges for next four days.

x + 4y = 22

(iii) $x + 4y = 22$ .....(i)

$x + 2y = 16$ .....(ii)

On solving (i) and (ii)

Therefore, additional charges is y = Rs.3

OR

For two more days price charged will be

$2y = 2 \times 3 = 6$

Total money paid by Amruta and Radhika is 22 + 16 + 6 + 6 = Rs.50

37. (i) Number of rose plants = 135

Number of marigold plants = 225

The maximum number of columns in which they can be planted

= HCF of 135 and 225

$\therefore$ Prime factors of $135 = 3 \times 3 \times 3 \times 5$

and $225 = 3 \times 3 \times 5 \times 5$

$\therefore$ Maximum number of columns = HCF $(135, 225) = 3 \times 3 \times 5 \times 5$

OR

Total number of plants 135 + 225 = 360 plants

(ii) We have proved that the maximum number of columns = 45

So, prime factors of $45 = 3 \times 3 \times 5$

$= 3^2 \times 5^1$

$\therefore$ Sum of exponents = 2 + 1 = 3.

(iii) Number of rows of Rose plants = $\frac{135}{45} = 3$

Number of rows of marigold plants = $\frac{225}{45} = 5$

Total number of rows = 3 + 5 = 8

38. (i) Area of grass field = $15 \times 15 = 225 \text{ m}^2$

(ii) Area of field horse can graze = $\frac{1}{4}\pi 5^2$

$= \frac{1}{4} \times \frac{22}{7} \times 25$

$= 19.64 \text{ m}^2$.

(iii) If rope was 10 m of grazing field

$= \frac{1}{4} \times \frac{22}{7} \times 100 = 78.57 \text{ m}^2$

OR

Increase in area = $78.57 - 19.64 = 58.93 \text{ m}^2$