MATHEMATICS SAMPLE PAPER # 04

ANSWER AND SOLUTIONS

SECTION-A

1. Option (1)

165

2. Option (3)

20

3. Option (1)

All real values except 10

4. Option (4)

Not defined

5. Option (2)

12

6. Option (3)

$\frac{11}{36}$

7. Option (4)

IV quadrant

8. Option (3)

4

9. Option (1)

–12

10. Option (2)

$\pi r(\ell + 2h + r)$

11. Option (4)

4

12. Option (1)

14, 38

13. Option (3)

$\frac{3}{11}$

14. Option (2)

5

15. Option (3)

12 cm

16. Option (2)

2r

17. Option (2)

2 : 3

18. Option (3)

±3

19. Option (2)

Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

20. Option (3)

Assertion (A) is true but Reason (R) is false.

SECTION-B

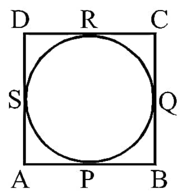
21. 110, 120, 130, ..., 990

$a_n = 990$

$\Rightarrow 110 + (n - 1) \times 10 = 990$

$\therefore n = 89$

22.



AP = AS, BP = BQ, CR = CQ and DR = DS

$\Rightarrow AP + BP + CR + DR = AS + BQ + CQ + DS$

$\Rightarrow AB + CD = AD + CB$

But AB = CD and AD = CB

$\therefore AB = AD$

Hence, ABCD is a square.

OR

Length of tangent $= 2 \times \sqrt{5^2 - 4^2} = 2 \times 3 \text{ cm} = 6 \text{ cm}$

23. $\Delta ADE \sim \Delta GBD$ and $\Delta ADE \sim \Delta FEC$

$\Rightarrow \Delta GBD \sim \Delta FEC$ (AA Criterion)

$\Rightarrow \frac{GD}{FC} = \frac{GB}{FE} \Rightarrow GD \times FE = GB \times FC$

or $FG^2 = BG \times FC$ [$\because GD = FE = FG$]

Hence proved

24. Capacity of first glass = $\pi r^2 H - \frac{2}{3}\pi r^3$

$= \pi \times 9\ (10 - 2) = 72\pi cm^3$

Capacity of second glass = $\pi r^2 H - \frac{1}{3}\pi r^2 h$

$= \pi \times 3 \times 3\ (10 - 0.5) = 85.5\pi cm^3$

$\therefore$ Suresh got more quantity of juice.

Extra amount = $13.5\ \pi cm^3$

25. For Jayanti,

Favourable outcome is (6, 6) i.e, 1

Probability (getting the number 36) = $\frac{1}{36}$

For Pihu,

Favourable outcome is 6 i.e, 1

Probability (getting the number 36) = $\frac{1}{6}$

$\therefore$ Pihu has the better chance.

OR

Total number of integers = 29

(i) Prob. (Prime number) = $\frac{6}{29}$

(ii) Prob (Number divisible by 7) = $\frac{4}{29}$

SECTION-C

26. Let us assume to the contrary, that $2\sqrt{5} - 3$ is a rational number

$\therefore\ 2\sqrt{5} - 3 = \frac{p}{q}$, where p and q are integers and $q \neq 0$

$\Rightarrow \sqrt{5} = \frac{p+3q}{2q}$ ..... (1)

Since p and q are integers $\therefore \frac{p+3q}{2q}$ is a rational number.

$\therefore\ \sqrt{5}$ is a rational number which is a contradiction as $\sqrt{5}$ is an irrational number.

Hence our assumption is wrong and hence $2\sqrt{5} - 3$ is an irrational number.

OR

$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$

$180 = 2 \times 2 \times 3 \times 3 \times 5$

HCF = $2 \times 2 \times 3 \times 3 = 36$

$13m - 16 = 36$

$13m = 52$

$m = 4$

27. $x + y = 7$ and $2(x - y) + x + y + 5 + 5 = 27$

$\therefore\ x + y = 7$ and $3x - y = 17$

Solving, we get, $x = 6$ and $y = 1$

28. (i) A(1, 7), B(4, 2), C(–4, 4)

Distance travelled by Seema = $\sqrt{34}$ units

Distance travelled by Aditya = $\sqrt{68}$ units

$\therefore$ Aditya travels more distance

(ii) Coordinate of D are

$$\left(\frac{1+4}{2}, \frac{7+2}{2}\right) = \left(\frac{5}{2}, \frac{9}{2}\right)$$

29. $\sin\theta + \cos\theta = \sqrt{3} \Rightarrow (\sin\theta + \cos\theta)^2 = 3$

$\Rightarrow 1 + 2\sin\theta\cos\theta = 3 \Rightarrow \sin\theta\cos\theta = 1$

$\therefore\ \tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = 1$

Hence proved

30. Required Area = Area of triangle - Area of 3 sectors

Area of Triangle = $\frac{1}{2} \times 24 \times 7 = 84\ m^2$

Area of three sectors

$= \frac{\pi r^2}{360°} \times$ (sum of three angles of triangle)

$= \frac{22 \times 7 \times 7 \times 180°}{7 \times 2 \times 2 \times 360°} = \frac{77}{4}$ or $19.25\ m^2$

$\therefore$ Required Area = $\frac{259}{4}$ or $64.75\ m^2$

OR

Quantity of water flowing through pipe in 1 hour

$= \pi \times \frac{7}{100} \times \frac{7}{100} \times 15000 m^3$

Required time

$= \left(50 \times 44 \times \frac{21}{100}\right) \div (\pi \times \frac{7}{100} \times \frac{7}{100} \times 15000)$

= 2 hours

31. LHS : $\frac{\frac{\sin^3\theta}{\cos^3\theta}}{1+\frac{\sin^2\theta}{\cos^2\theta}} + \frac{\frac{\cos^3\theta}{\sin^3\theta}}{1+\frac{\cos^2\theta}{\sin^2\theta}}$

$= \frac{\frac{\sin^3\theta}{\cos^3\theta}}{\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}} + \frac{\frac{\cos^3\theta}{\sin^3\theta}}{\frac{\sin^2\theta+\cos^2\theta}{\sin^2\theta}}$

$= \frac{\sin^3\theta}{\cos\theta} + \frac{\cos^3\theta}{\sin\theta}$

$= \frac{\sin^4\theta+\cos^4\theta}{\cos\theta\sin\theta}$

$= \frac{(\sin^2\theta+\cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta}{\cos\theta\sin\theta}$

$= \frac{1-2\sin^2\theta\cos^2\theta}{\cos\theta\sin\theta}$

$= \frac{1}{\cos\theta\sin\theta} - \frac{2\sin^2\theta\cos^2\theta}{\cos\theta\sin\theta}$

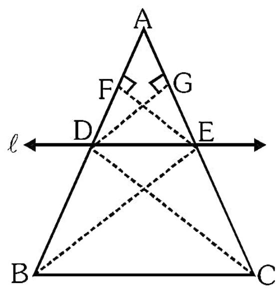
= $\sec\theta\ \text{cosec}\theta - 2\sin\theta\cos\theta$

= RHS

SECTION-D

32. **Given :** A ΔABC in which line ℓ parallel to BC (DE$\parallel$BC) intersecting AB at D and AC at E.

To prove : $\frac{AD}{DB} = \frac{AE}{EC}$



Construction : Join D to C and E to B. Through E draw EF perpendicular to AB i.e., EF $\perp$ AB and through D draw DG $\perp$ AC.

Proof :

Area of (ΔADE) = $\frac{1}{2}$(AD $\times$ EF) ...(1)

(Area of $\Delta = \frac{1}{2}$ base $\times$ altitude)

Area of (ΔBDE) = $\frac{1}{2}$(BD $\times$ EF) ...(2)

Dividing (1) by (2)

$\frac{\text{Area }(\Delta ADE)}{\text{Area }(\Delta BDE)} = \frac{\frac{1}{2}AD \times EF}{\frac{1}{2}BD \times EF} = \frac{AD}{DB}$ ...(3)

Similarly, $\frac{\text{Area }(\Delta ADE)}{\text{Area }(\Delta CDE)} = \frac{\frac{1}{2}AE \times DG}{\frac{1}{2}EC \times DG} = \frac{AE}{EC}$

$$\frac{\text{Area }(\Delta ADE)}{\text{Area }(\Delta CDE)} = \frac{AE}{EC} \quad ...(4)$$

Area (ΔBDE) = Area (ΔCDE) ...(5)

[Δs BDE and CDE are on the same base DE and between the same parallel lines DE and BC.]

From (4) and (5)

$$\frac{\text{Area}(\Delta ADE)}{\text{Area}(\Delta BDE)} = \frac{AE}{EC} \quad ...(6)$$

From (3) and (6)

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Hence proved.

33. Let the original speed of the train be x km/h

$$\therefore \quad \frac{360}{x} - \frac{360}{x+5} = \frac{48}{60}$$

$\Rightarrow \quad x^2 + 5x - 2250 = 0$

$\Rightarrow \quad (x + 50)(x - 45) = 0$

$\therefore \quad x = 45$

Hence original speed of the train = 45km/h

OR

$$\frac{1}{x} - \frac{1}{x-2} = 3$$

$$\frac{x-2-x}{x(x-2)} = \frac{3}{1}$$

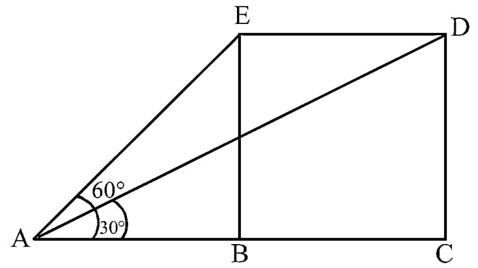
$3x^2 - 6x = -2$

$3x^2 - 6x + 2 = 0$

$$x = \frac{6 \pm \sqrt{12}}{6}$$

$$= \frac{3+\sqrt{3}}{3}, \frac{3-\sqrt{3}}{3}$$

34.



In ΔABE, $\frac{BE}{AB} = \tan 60°$

$\Rightarrow$ AB = 3000 m

In ΔDAC, $\frac{DC}{AC} = \tan 30°$

$\Rightarrow$ AC = 9000 m

BC = AC – AB = 6000m

$\therefore$ Speed of aeroplane = $\frac{6000}{30}$ m/s = 200m/s

= 720 km/hr

35.

Daily Wages (in Rs.)	Number of Workers(f_i)	x_i	u_i	f_iu_i
100-120	10	110	– 3	– 30
120-140	15	130	– 2	– 30
140-160	20	150	– 1	– 20
160-180	22	170	0	0
180-200	18	190	1	18
200-220	12	210	2	24
220-240	13	230	3	39
Total	110			1

Mean daily wages = 170 + $\frac{1}{110}$×20 = ₹170.19 (approx.)

Mode = 160 + $\frac{22-20}{44-20-18}$ × 20 = ₹166.67 (approx.)

OR

Re-writing the distribution in the form of the grouped distribution with each class interval as 10 and taking assumed mean to be 55, we get the following table.

Class	Mid – value $\left(x_i = \frac{\ell + u}{2}\right)$	$d_i = x_i - A$ (A = 55)	$u_i = \frac{d_i}{h}$	Number of students (f_i)	f_iu_i
0 – 10	5	–50	–5	12	–60
10 – 20	15	–40	–4	10	–40
20 – 30	25	–30	–3	13	–39
30 – 40	35	–20	–2	15	–30
40 – 50	45	–10	–1	20	–20
50 – 60	55 = A	0	0	16	0
60 – 70	65	10	1	11	11
70 – 80	75	20	2	7	14
80 – 90	85	30	3	5	15
90 – 100	95	40	4	6	24

$$\text{Mean} = A + \frac{f_iu_i}{f_i} \times h = 55 + \frac{-125}{115} \times 10$$

= 44.13 (approx)

SECTION-E

36. (i) Since each row is increasing by 10 seats, so it is an AP with first term a = 30, and common difference d = 10.

So number of seats in 10^{th} row

$= a_{10}$

= a + 9d

= 30 + 9 × 10 = 120

(ii) $S_n = \frac{n}{2}[2 \times 30 + (n - 1)10]$

$1500 = \frac{n}{2}[2 \times 30 + (n - 1)10]$

$3000 = 50n + 10n^2$

$n^2 + 5n - 300 = 0$

$n^2 + 20n - 15n - 300 = 0$

(n + 20)(n – 15) = 0

Rejecting the negative value, n = 15

OR

Number of seats already put up to the 10^{th} row = S_{10}

$S_{10} = \frac{10}{2}\{(2 \times 30 + (10 - 1)10)\}$

= 5(60 + 90) = 750

So, the number of seats still required to be put are 1500 – 750 = 750

(iii) If number of rows = 17

then the middle row is the 9^{th} row

$a_8 = a + 8d = 30 + 80 = 110$ seats

37. (i) Let AD be x cm, then DB = (12 – x) cm

∵ AD = AF, CF = CE, DB = BE

[tangents to a circle from an external point]

∴ AF = xcm,

then CF = (10 – x)cm

BE = (12 – x)cm,

then CE = 8 – (12 – x) = (x – 4) cm

Now CF = CE

10 – x = x – 4

2x = 14

⇒ x = 7

Hence, AD = 7 cm

Since, ∵ BE = (12 – x)cm = (12 – 7) cm

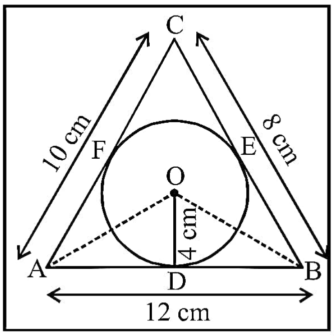
[∵ x = 7 proved]

BE = 5 cm

(ii) Radius, OD = 4 cm

and AB = 12 cm

Then, area of ΔO



Then, area of ΔOAB

$= \frac{1}{2} \times OD \times AB$

$= \frac{1}{2} \times 4 \times 12$

$= 24\ cm^2$

(iii) Perimeter of ΔABC = AB + BC + CA

= (12 + 8 + 10) cm

= 30 cm

OR

Since, 100 cm cost = Rs.1500

So, 30 cm cost = $\frac{1500 \times 30}{100}$ = Rs.450

38. (i) For cuboid

ℓ = 15 cm, b = 10 cm and h = 3.5 cm

Volume of the cuboid = $\ell \times b \times h$

$= 15 \times 10 \times 3.5$

$= 525\ cm^3$

(ii) For conical depression :

r = 0.5 cm,

h = 1.4 cm

Volume of conical depression

$= \frac{1}{3} \times \frac{22}{7} \times 0.5 \times 0.5 \times 1.4$

$= \frac{11}{30}\ cm^3$

(iii) Volume of four conical depressions

$= 4 \times \frac{11}{30} = 1.47\,cm^3$

OR

Volume of the wood in the entire stand

= Volume of cuboid – Volume of 4 conical depressions

= 525 – 1.47

$= 523.53\ cm^3$