PAPER # 02 - MATHEMATICS

1. (a) The probability of an impossible event is 0.

2. (c) On a die, there are six numbers 1,2,3,4,5 and 6.

∴ Total number of possible outcomes = 6
Number on dice which are greater than 4 = 5, 6

∴ Favourable number of elementary events = 2

∴ Required probability = $\frac{2}{6} = \frac{1}{3}$

3. (b) We have, $\sin^2\theta + \frac{1}{1+\tan^2\theta}$

$= \sin^2\theta + \frac{1}{\sec^2\theta}$ [∵ $\sec^2 A = 1 + \tan^2 A$]

$= \sin^2\theta + \cos^2\theta$ [∵ $\sec A = \frac{1}{\cos A}$]

$= 1$ [∵ $\sin^2 A + \cos^2 A = 1$]

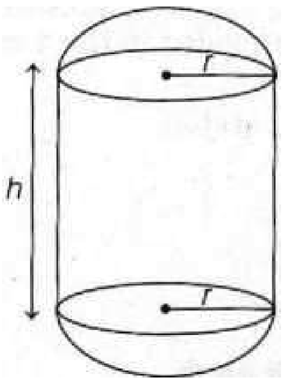
4. (d) We know that area of sector A of radius r and length of arc / is given by

$A = \frac{1}{2} lr$

$\therefore A = \frac{1}{2} \times 3.5 \times 5$

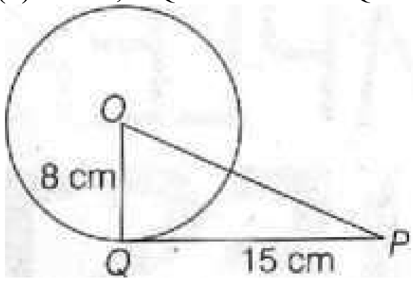
$= 8.75 cm^2$

5. (c)



Total curved surface area
= Curved surface area of cylinder
+ 2 × Curved surface area of hemispheres
$= 2\pi rh + 2 \times (2\pi r^2)$
$= 2\pi rh + 4\pi r^2$

6. (b) Given, OQ = 8 cm and PQ = 15 cm.



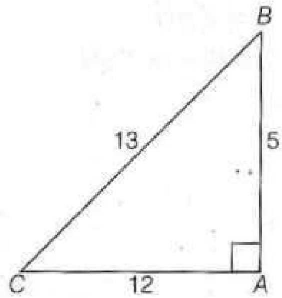
In right angled ΔOPQ, using Pythagoras theorem

$OP = \sqrt{OQ^2 + QP^2} = \sqrt{8^2 + 15^2}$

$= \sqrt{64 + 225} = \sqrt{289} = 17$ cm

7. (c) $\frac{1}{2}$ [upper limit + lower limit]

8. (c) With reference to ∠B, we have Base = AB = 5, perpendicular = AC = 12 and hypotenuse = BC =13



$\therefore \sin B = \frac{AC}{BC} = \frac{12}{13}$

and $\tan B = \frac{AC}{AB} = \frac{12}{5}$

9. (b) $3 \sin 30° - 4\sin^3 60°$

$= 3 \times \frac{1}{2} - 4\left(\frac{\sqrt{3}}{2}\right)^3 = \frac{3}{2} - 4 \times \frac{3\sqrt{3}}{8}$

$= \frac{3}{2} - \frac{3\sqrt{3}}{2} = \frac{3 - 3\sqrt{3}}{2} = \frac{3\left(1-\sqrt{3}\right)}{2}$

10. (c)

11. **(d)** The given equations can be re-written as

$4x + ky - 1 = 0$ and $6x - 10y - 14 = 0$

On comparing with $a_1x + b_1y + c_1 = 0$ and

$a_{2x} + b_2y + c_2 = 0$, we get

$a_1 = 4,\ b_1 = k, c_1 = -1$

and $a_2 = 6,\ b_2 = -10,\ c_2 = -14$

For unique solution,

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \quad \Rightarrow \frac{4}{6} \neq \frac{k}{-10}$

$\Rightarrow k \neq -\frac{20}{3}$

Thus, given lines have a unique solution for all real values of k, except $-\frac{20}{3}$.

12. (c) Given, AP is 21, 18, 15, ...

Here, a = 21 and d = 18 – 21 = –3

Let n th term of given AP be –81.

Then, $a_n = -81$

$\Rightarrow a + (n-1)d = -81 \left[\because a_n = a + (n-1)d\right]$ ...(i)

On putting the values of a and d in Eq. (i), we get

$21 + (n - 1) \times (-3) = -81$

$\Rightarrow 21 - 3n + 3 = -81$

$\Rightarrow 24 - 3n = -81$

$\Rightarrow -3n = -81 - 24$

$\therefore n = \frac{-105}{-3} = 35$

Hence, 35th term of given AP is -81.

13. (c) AB = $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

14. (b) Given, a = 2, a_{20} = 62 and n = 20

Now, sum of first 20 terms

$S_{20} = \frac{20}{2}(2 + 62)$ [$\because S_n = \frac{n}{2}(a + a_n)$]

= 10 × 64 = 640

15. (d) HCF (a, b) = 1

LCM (a, b) = ab

∴ HCF (a, b) × LCM (a, b) = 1 × ab = ab

16. (b) Let α and β be the zeroes of $(mx^2 - 6x - 6)$. Here, a = m, b = –6 and c = –6

Given, αβ = – 3

$\therefore \frac{c}{a} = -3 \Rightarrow \frac{-6}{m} = -3 \Rightarrow m = 2$

17. (a) Given, $2x^2 - 5x - 3 = 0$

Splitting the middle term, we get

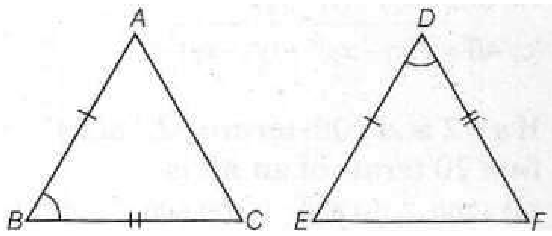
$2x^2 - 6x + x - 3 = 0$

$\Rightarrow 2x(x - 3) + 1(x - 3) = 0$

$\Rightarrow (x - 3)(2x + 1) = 0$

$\Rightarrow x = -\frac{1}{2}, 3$

18. (c) Given, in ΔABC and ΔDEF, $\frac{AB}{DE} = \frac{BC}{FD}$



ΔABC and ΔEDF will be similar, if

∠B = ∠D [by SAS similarity criterion]

19. (c) Assertion sin 60°cos 30° + sin 30°cos 60°

$\frac{\sqrt{3}}{2}\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$

So, Assertion is true.

Reason We know, sin 90° = 1 and cos 90° = 0

So, Reason is false.

Assertion (A) is true but Reason (R) is false

20. **(a) Assertion (A)**

Here, $a_1 = 2, b_1 = 3, c_1 = 5$

and $a_2 = 4, b_2 = 6, c_2 = 7$ [∵ k = 6]

So, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \left[\because \frac{2}{4} = \frac{3}{6} \neq \frac{5}{7}\right]$

So, the given system of equations has no solution (i.e. inconsistent).

So, the Assertion is true.

Reason (R) $a_1x + b_1y + c_1 = 0$

and $a_2x + b_2y + c_2 = 0$

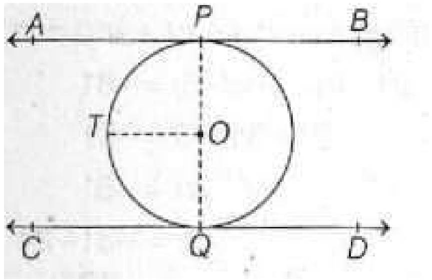
We know, for the system of equations to be inconsistent,

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

So, both Assertion and Reason are true and Reason is a correct explanation of Assertion.

21. Given AB and CD are two parallel tangents at the point P and Q of a circle with centre 0.

To prove POQ is a diameter of the circle.



Construction Join OP and OQ and draw OT || AS.

Proof ∠APO + ∠TOP = 180° [∵ OT || AB]

⇒ 90° + ∠TOP = 180° [∵ OP 1 AB]

⇒ ∠TOP = 180° - 90° = 90°

Similarly, ∠TOQ = 90°

∴ ∠TOP + ∠TOQ = 90° + 90° = 180°

Since, POQ is a straight line.

Hence, POQ is a diameter of the circle with centre O. **Hence proved.**

22. Out of 25 numbers, 1,2,3,…, 25 one number can be chosen in 25 ways.

∴ Total number of elementary events = 25 (1)

The number selected will be a prime number, if it is chosen from the numbers 2, 3, 5, 7, 11, 13, 17, 19, 23.

∴ Favorable number of elementary events = 9

Hence, required probability = $\frac{9}{25}$ (1)

Or

There are 13 letters in the word 'ASSASSINATION' out of which one letter can be chosen in 13 ways.

∴ Total number of elementary events = 13

(i) There are 6 vowels in the word 'ASSASSINATION'. So, there are 6 ways of selecting a vowel.

Probability of selecting a vowel = $\frac{6}{13}$ (1)

(ii) We have, probability of selecting a consonant

= 1 – Probability of selecting a vowel

$= 1 - \frac{6}{13} = \frac{7}{13}$ (1)

23. The sequence goes like this

2, 4, 6, 8, ...

Here, 4 – 2 = 6 – 4 = 8 – 6 = 2

So, it is an AP with first term, a = 2,

common difference, d = 4 – 2 = 2

and total number of terms, n = 15 (1)

∴ Sum of first 15 even natural numbers

$S_{15} = \frac{n}{2}[2a + (n-1)d] = \frac{15}{2}[2 \times 2 + (15-1)2]$

$\left[\because S_n = \frac{n}{2}\{2a + (n-1)d\}\right]$

$= \frac{15}{2}[4+28] = \frac{15}{2} \times 32 = 240$ (1)

24. Since, x = 2 is a root of the equation $2x^2 + kx - 6 = 0$

$\therefore 2 \times 2^2 + 2k - 6 = 0$

$\Rightarrow 8 + 2k - 6 = 0$

$\Rightarrow 2k + 2 = 0 \Rightarrow k = -1$ (1)

On putting k = –1 in the equation $2x^2 + kx - 6 = 0$, we get

$2x^2 - x - 6 = 0 \Rightarrow 2x^2 - 4x + 3x - 6 = 0$

$\Rightarrow 2x(x-2) + 3(x-2) = 0 \Rightarrow (x-2)(2x+3) = 0$

$\Rightarrow x - 2 = 0$ or $2x + 3 = 0$

$\Rightarrow x = 2$ or $-\frac{3}{2}$

Hence, the other root is $-\frac{3}{2}$. (1)

25. LHS = cot A + tan A = $\frac{\cos A}{\sin A} + \frac{\sin A}{\cos A}$

$\left[\because \cot\theta = \frac{\cos\theta}{\sin\theta}, \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$

$= \frac{\cos^2 A + \sin^2 A}{\sin A \cdot \cos A} = \frac{1}{\cos A \cdot \sin A}$

$\left[\because \sin^2\theta + \cos^2\theta = 1\right]$ (1)

$= \frac{1}{\sin A} \cdot \frac{1}{\cos A} =$ cosec A sec A

$\left[\because \operatorname{cosec}\theta = \frac{1}{\sin\theta} \text{ and } \sec\theta = \frac{1}{\cos\theta}\right]$

RHS Hence proved. (1)

Or

We have, $\cos^2 30° + \sin^2 45° - \frac{1}{3}\tan^2 60°$

$= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 - \frac{1}{3}\left(\sqrt{3}\right)^2$ (1)

$$\left[\because \cos 30^\circ = \frac{\sqrt{3}}{2} \sin 45^\circ = \frac{1}{\sqrt{2}} \text{ and } \tan 60^\circ = \sqrt{3}\right]$$

$$= \frac{3}{4} + \frac{1}{2} - \frac{3}{3} = \frac{3+2}{4} - 1 = \frac{5}{4} - 1 = \frac{5-4}{4} \equiv \frac{1}{4} \ (1)$$

26. **Given** in figure, two chords AS and CD intersect each other at point P.

To prove (i) $\Delta APC \sim \Delta DPB$

(ii) $AP \cdot PB = CP \cdot DP$

Proof (i) In ΔAPC and ΔDPB,

$\angle APC = \angle DPB$ [vertically opposite angles]

and $\angle CAP = \angle BDP$

[angles in the same segment]

$\therefore$ $\Delta APC \sim \Delta DPB$ [by AA similarity criterion] (1)

(ii) We have, $\Delta APC \sim \Delta DPB$ [proved in part (i)]

$\therefore \frac{AP}{DP} = \frac{CP}{BP}$ (1)

[$\because$ if two triangles are similar, then the ratio of their corresponding sides is equal]

$\therefore AP \cdot BP = CP \cdot DP$

or AP-PB = CP-DP Hence proved. (1)

27. Given, a circle is inscribed in the triangle, whose sides are BC = 8cm, AC = 10 cm and AS = 12 cm.

Let AD = AF = x, BD = BE = y

and CE = CF = z

[$\because$ the length of two tangents drawn from an external point to a circle are equal]

We have, AB = 12

$\Rightarrow$ AD + DB = 12 $\Rightarrow$ x + y = 12 ...(i)

AC = 10 $\Rightarrow$ AF + FC = 10

$\Rightarrow$ x + z – 10 ...(ii) and BC = 8

$\Rightarrow$ CE + EB = 8 $\Rightarrow$ z + y = 8 ..(iii)(1)

On adding Eqs. (i), (ii) and (iii), we get

2(x + y + z) = 12 + 10 + 8

$\Rightarrow x + y + z = \frac{30}{2} = 15$ ...(iv)

On putting x + y = 12 from Eq. (i) in Eq. (iv), we get

12 + z = 15

$\Rightarrow$ z = 3

On putting z + y = 8 from Eq. (iii) in Eq, (iv), we get

x + 8 = 15

$\Rightarrow$ x = 7 (1)

On putting x + z = 10 from Eq, (ii) in Eq. (iv), we get

10 + y = 15

$\Rightarrow$ y = 5

Hence, AD = 7 cm, BE = 5 cm and CF = 3cm (1)

28. $\text{LHS} = \frac{\sin A + \cos A}{\sin A - \cos A} + \frac{\sin A - \cos A}{\text{Sin} A + \cos A}$

$$= \frac{(\sin A + \cos A)^2 + (\sin A - \cos A)^2}{(\sin A - \cos A)(\sin A + \cos A)} \ (1)$$

$$= \frac{[\sin^2 A + 2\sin A \cos A + \cos^2 A + \sin^2 A - 2\sin A \cos A + \cos^2 A}{\sin^2 A - \cos^2 A}$$

$\left[\because (a \pm b)^2 = a^2 + b^2 \pm 2ab\right]$ (1)

$$= \frac{2\sin^2 A + 2\cos^2 A}{\sin^2 A - \cos^2 A} = \frac{2(\sin^2 A + \cos^2 A)}{\sin^2 A - \cos^2 A}$$

$$= \frac{2}{\sin^2 A - \cos^2 A} \ [\because \sin^2\theta + \cos^2\theta = 1]$$

= RHS Hence proved. (1)

Or

$$\text{LHS} = \frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{\tan\theta - 1 + \sec\theta}{\tan\theta + 1 - \sec\theta}$$

[dividing numerator and denominator by $\cos\theta$]

$$= \frac{(\tan\theta + \sec\theta) - 1}{(\tan\theta - \sec\theta) + 1}$$

$$= \frac{[(\tan\theta + \sec\theta) - 1][\tan\theta - \sec\theta]}{[(\tan\theta - \sec\theta) + 1][\tan\theta - \sec\theta]}$$

[multiplying and dividing by ($\tan\theta - \sec\theta$)] (1)

$$= \frac{(\tan^2\theta - \sec^2\theta) - (\tan\theta - \sec\theta)}{(\tan\theta - \sec\theta + 1)(\tan\theta - \sec\theta)}$$

[$\because (a - b)(a + b) = a^2 - b^2$]

$$= \frac{-1 - \tan\theta + \sec\theta}{(\tan\theta - \sec\theta + 1\rangle(\tan\theta - \sec\theta)}$$

[$\because \tan^2 A - \sec^2 A = -1$] (1)

$$= \frac{-(\tan\theta - \sec\theta + 1)}{(\tan\theta - \sec\theta + 1)(\tan\theta - \sec\theta)}$$

$$= \frac{-1}{\tan\theta - \sec\theta}$$

$$= \frac{1}{\sec\theta - \tan\theta} = \text{ RHS Hence proved. (1)}$$

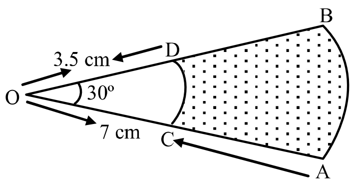
29. Let A_1 and A_2 be the areas of sectors OAB and OCD respectively. Then, A_1 = Area of a sector of angle 30º in a circle of radius 7 cm

$$\Rightarrow A_1 = \left\{\frac{30}{360} \times \frac{22}{7} \times 7^2\right\} \text{ cm}^2$$

$$\left[\text{Using: } A = \frac{\theta}{360} \times \pi r^2\right]$$

$\Rightarrow A_1 = 77/6\ \text{cm}^2$

A_2 = Area of a sector of angle 30º in a circle of radius 3.5 cm.



$$\Rightarrow A_2 = \left\{\frac{30}{360} \times \frac{22}{7} \times (3.5)^2\right\} \text{cm}^2$$

$$\Rightarrow A_2 = \left\{\frac{1}{12} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\right\} \text{cm}^2 = \frac{77}{24} \text{cm}^2$$

$\therefore$ Area of the shaded region

$$= A_1 - A_2 = \left(\frac{77}{6} - \frac{77}{24}\right) \text{cm}^2$$

$$= \frac{77}{24} \times (4 - 1) \text{ cm}^2 = 77/8 \text{ cm}^2 = 9.625 \text{ cm}^2$$

30. Given equation is

$$\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30},\ x \neq -4, 7$$

$$\Rightarrow \frac{(x-7)-(x+4)}{(x+4)(x-7)} = \frac{11}{30}$$

$$\Rightarrow \frac{x-7-x-4}{x^2-7x+4x-28} = \frac{11}{30}$$

$$\Rightarrow \frac{-11}{x^2-3x-28} = \frac{11}{30}$$

$$\Rightarrow \frac{-1}{x^2-3x-28} = \frac{1}{30}$$

$\Rightarrow -30 = x^2 - 3x - 28$

$\Rightarrow x^2 - 3x + 2 = 0$ (1)

On comparing with the standard quadratic equation $ax^2 + bx + c = 0$, we get

$a = 1,\ b = -3$ and $c = 2$

By using quadratic formula, we get

$$x = \frac{-b \pm \sqrt{2^2 - 4ac}}{2a}$$

$$= \frac{-(-3) \pm \sqrt{(3)^2 - 4(1)(2)}}{2 \times 1} \text{ (1)}$$

$$= \frac{3 \pm \sqrt{9-8}}{2} = \frac{3 \pm \sqrt{1}}{2} = \frac{3 \pm 1}{2}$$

$$\Rightarrow x = \frac{3+1}{2} \text{ or } x = \frac{3-1}{2} x = \frac{4}{2} \text{ or } x = \frac{2}{2}$$

$\therefore$ $x = 2$ or $x = 1$

Hence the roots of the given equation are 2 and 1. (1)

Or

Let $\frac{2x+3}{x-3} = y$ …(i)

Then, $\frac{x-3}{2x+3} = \frac{1}{y}$ (1/2)

Therefore, the given equation reduces to

$$2y - 25\frac{1}{y} = 5$$

$\Rightarrow 2y^2 - 25 = 5y$

$\Rightarrow 2y^2 - 5y - 25 = 0$

$\Rightarrow 2y^2 - 10y + 5y - 25 = 0$ [by factorisation method]

$\Rightarrow 2y(y–5) + 5(y – 5) = 0$

$\Rightarrow (y – 5)(2y + 5) = 0$

$\Rightarrow y = 5$ or $y = \frac{-5}{2}$ (1)

Now, putting y = 5 in Eq. (i), we get

$$\frac{2x+3}{x-3} = \frac{5}{1}$$

$\Rightarrow 5x – 15 = 2x + 3$

$\Rightarrow 3x = 18$

$\Rightarrow x = 6$ (1/2)

Again, putting $y = \frac{5}{2}$ in Eq. (i), we get

$\frac{2x+3}{x-3} = \frac{5}{1}$

$\Rightarrow 5x + 15 = 4x + 6$

$\therefore 9x = 9$

$\Rightarrow x = 1$

Hence, the values of x are 1 and 6. (1)

31. The given system of linear equations can be written as

$2x + 3y - 7 = 0$

$(a - b)x + (a + b)y - (3a + b - 2) = 0$

The above system of equations is of the form

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0,$

where $a_1 = 2, b_1 = 3, c_1 = -7$

$a_2 = (a - b), b_2 = (a + b), c_2 = -(3a + b - 2)$

For the given system of equations to have an infinite number of solutions

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Here, $\frac{a_1}{a_2} = \frac{2}{a-b}, \frac{b_1}{b_2} = \frac{3}{a+b}$ and

$\frac{c_1}{c_2} = \frac{-7}{-(3a+b-2)} = \frac{7}{3a+b-2}$

$\Rightarrow \frac{2}{a-b} = \frac{3}{a+b} = \frac{7}{3a+b-2}$

$\Rightarrow \frac{2}{a-b} = \frac{3}{a+b}$ and $\frac{3}{a+b} = \frac{7}{3a+b-2}$

$\Rightarrow 2a + 2b = 3a - 3b$ and $9a + 3b - 6 = 7a + 7b$

$\Rightarrow 2a - 3a = -3b - 2b$ and $9a - 7a = 7b - 3b + 6$

$\Rightarrow -a = -5b$ and $2a = 4b + 6$

$\Rightarrow a = 5b$ .... (3) and $a = 2b + 3$ .... (4)

Solving (3) and (4) we get

$5b = 2b + 3 \Rightarrow b = 1$

Substituting b = 1 in (3), we get

$a = 5 \times 1 = 5$

Thus, a = 5 and b = 1

Hence, the given system of equations has infinite number of solutions when

a = 5, b = 1

32. Let us assume, to the contrary, that $\sqrt{2}$ is rational.

So, we can find integers r and s ($\neq 0$) such that $\sqrt{2} = \frac{r}{s}$.

Suppose r and s not having a common factor other than 1. Then, we divide by the common factor to get $\sqrt{2} = \frac{a}{b}$, where a and b are coprime.

So, $b\sqrt{2} = a$.

Squaring on both sides and rearranging, we get $2b^2 = a^2$. Therefore, 2 divides a^2. Now, by Theorem it following that 2 divides a.

So, we can write a = 2c for some integer c.

Substituting for a, we get $2b^2 = 4c^2$, that is, $b^2 = 2c^2$.

This means that 2 divides b^2, and so 2 divides b (again using Theorem with p = 2).

Therefore, a and b have at least 2 as a common factor.

But this contradicts the fact that a and b have no common factors other than 1.

This contradiction has arisen because of our incorrect assumption that $\sqrt{2}$ is rational.

So, we conclude that $\sqrt{2}$ is irrational.

Or

The number of participants in each room must be the HCF of 60, 84 and 108. (1)

Now, prime factors of numbers 60, 84 and 108 are

$60 = 2^2 \times 3 \times 5,$

$84 = 2^2 \times 3 \times 7$

and $108 = 2^2 \times 3^3$

HCF of (60, 84, 108) = $2^2 \times 3 = 12$ (2)

Therefore, in each room maximum 12 participants can be seated,

$\therefore$ Total number of participants = 60 + 84 + 108 = 252

$\therefore$ Number of rooms required= $\frac{252}{12} = 21$ (2)

33. Table for cumulative frequency is given below

Class interval	Frequency	Cumulative frequency
0-6	4	4+0 = 4
6-12	X	4 + x = (4 + x) (c f)
12-18	5(f)	5 + (4 + x) = 9 + x
18-24	Y	y + (9 + x) = 9 + x + y
24-30	1	1 + (9 + x + y) = 10 + x + y

(1)

Since, N =20

$\therefore$ 10 + x + y = 20

$\Rightarrow$ x + y = 20 - 10

$\Rightarrow$ x + y = 10 ...(i)

Also, we have, median = 14.4

which lies in the class interval 12-18. (1)

$\therefore$ The median class is 12-18, such that

l = 12, f = 5, c f = 4 + x and h = 6

$$\therefore \text{Median} = l + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h$$

$$\Rightarrow 14.4 = 12 + \left[\frac{10-(4+x)}{5}\right]\times 6$$

$$\Rightarrow 14.4 - 12 = \frac{6-x}{5}\times 6$$

$$\Rightarrow 2.4 = \frac{36-6x}{5}$$

$\Rightarrow$ 12 = 36 – 6x

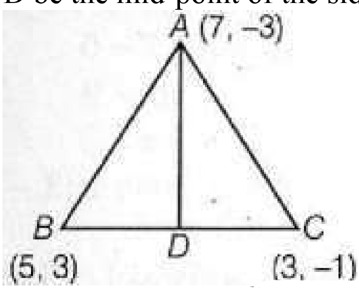
$\Rightarrow$ 6x = 24 $\Rightarrow$ x = 4 (1)

Now, put the value of x in Eq. (i), we get

4 + y = 10 $\Rightarrow$ y = 10 – 4 = 6

Thus, x = 4 and y = 6 (1)

34. The median from a vertex of a triangle bisects the opposite side, to that vertex. So, let AD be the median through A then D be the mid-point of the side BC.



Now, coordinates of D = $\left(\frac{5+3}{2}\frac{3-1}{2}\right) = (4, 1)$

[$\because$ coordinates of mid -point of line segment joining (x_1, y_1) and (x_2, y_2) = $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$]

and length of median AD is given by

$$AD = \sqrt{(x_2-x_1)^2+(y^2+y_1)^2}$$

[by distance formula] (1)

$$= \sqrt{(4-7)^2+(1+3)^2} = \sqrt{(-3)^2+(4)^2}$$

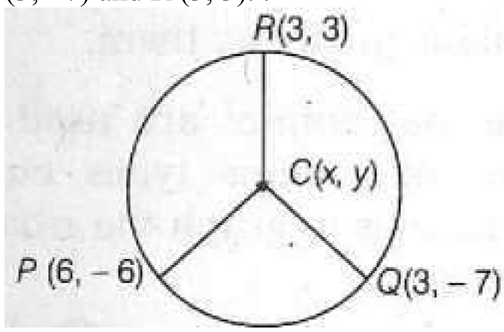
$\sqrt{9+16} = \sqrt{25}$ = 5 units (1)

Also, OA = $\sqrt{(0-7)^2+(0+3)^2}$

= $\sqrt{49+9} = \sqrt{58}$ units (1)

Or

Let C (x, y) be the centre of the circle passing through the points P (6, - 6), Q (3, - 7) and R (3, 3). .



(2)

Then, PC = QC = CP [radii of circle]

Now, PC = QC

$\Rightarrow$ $PC^2 = QC^2$ [squaring both sides]

$\Rightarrow$ $(x - 6)^2 + (y + 6)^2 = (x - 3)^2 + (y + 7)^2$

[$\because$ distance = $\sqrt{(x_2-x_1)^2+(y^2-y_1)^2}$]

$\Rightarrow$ $x^2 - 12.x + 36 + y^2 + 12y + 36$

$= x^2 - 6x + 9 + y^2 + 14y + 49$

[$\because$ $(a - b)^2 = a^2 + b^2 - 2ab$] (1)

$\Rightarrow$ –12x + 6x + 12y -14y + 72 - 58 = 0

$\Rightarrow$ – 6x – 2y + 14 = 0

$\Rightarrow$ 3x + y – 7 = 0 [dividing by – 2] ...(i)

and QC = CR

$\Rightarrow$ QC = CR [squaring both sides]

$\Rightarrow$ $(x - 3)^2 + (y + 7)^2 = (x - 3)^2 + (y - 3)^2$

$\Rightarrow$ $(y + 7)^2 = (y - 3)^2$ (1)

$\Rightarrow y^2 + 14y + 49 = y^2 - 6y + 9$
$\Rightarrow 20y + 40 = 0$
$\Rightarrow y = -\frac{40}{20} = -2$ ...(ii)
On putting y = – 2 in Eq. (i), we get
$3x - 2 - 7 = 0$
$\Rightarrow 3x = 9$
$\Rightarrow x = 3$
Hence, the centre of circle is (3,-2). (1)

35. Let BPC be the hemisphere and ABC be the cone standing on the base of the hemisphere (see figure). The radius BO of the hemisphere (as well as of the cone)
$= \frac{1}{2} \times 4$ cm = 2 cm

So, volume of the toy = $\frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$
$= \left[\frac{2}{3} \times 3.14 \times (2)^3 + \frac{1}{3} \times 3.14 \times (2)^2 \times 2\right] cm^3$
$= 25.12\ cm^3$
Now, let the right circular cylinder EFGH circumscribe the given solid. The radius of the base of the right circular cylinder = HP = BO = 2 cm, and its height is
EH = AO + OP = (2 + 2) cm = 4 cm
So, the volume required
= Volume of the right circular cylinder – volume of the toy
$= (3.14 \times 2^2 \times 4 - 25.12)\ cm^3$
$= 25.12\ cm^3$
$= 25.12\ cm^3$
Hence, the required difference of the two volumes = $25.12\ cm^3$.

36. (i) Sol: -2 and 8
(ii) Zeroes are – 2 and 8
$\therefore p(x) = k[x^2 - (\alpha + \beta)x + \alpha\beta]$
$= k[x^2 - (-2 + 8)x + (-2\times8)]$
$= k[x^2 - 6x - 16]$
But here parabola is inverted
$\therefore p(x) = -(x^2 - 6x - 16)$

(iii) $p(x) = -x^2 + 3x - 2$
$p(x) = 0$
$-x^2 + 3x - 2 = 0$
$x^2 - 3x + 2 = 0$
$x^2 - 2x - x + 2 = 0$
$x(x - 2) - 1(x - 2) = 0$
$(x - 2)(x - 1) = 0$
x =2, x =1

OR

(iii) $(x-4)(x+7) = x^2 + 3x - 28$

37. (i) To find the angle of elevation,
$\tan\theta = \frac{\text{height of tower}}{\text{distance from tower}} = \frac{42}{42} = 1$
$\theta = \tan^{-1}(1) = 45^o$

(ii) To find the distance,
$\tan 60^o = \frac{\text{height of tower}}{\text{distance}} = \frac{42}{\text{distance}}$
$\sqrt{3} = \frac{42}{\text{distance}}$
Distance $= \frac{42}{\sqrt{3}}$
Distance = 24.24 m

(iii) To find the height of the verticle tower,
$\tan 60^o = \frac{\text{height of the tower}}{\text{distance}}$
$\sqrt{3} = \frac{\text{height of the tower}}{20}$
Height of the tower = $20\sqrt{3}$

(iii) To find the angle of elevation of the sun,
$\tan\theta = \frac{\text{height of the tower}}{\text{distance from the tower}}$
$= \frac{1}{1}$ (since the ratios are in 1 : 1)
$\theta = \tan^{-1}(1) = 45^o$

38. (i) $6\sqrt{3}$
(ii) $308\ cm^2$
(iii) $1078\ cm^3$

OR

(iii) 25 cm