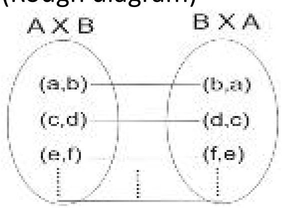
Class 12th
Relations & Functions

Q.1) Let A and B are any two-empty sets. Show that $f: A \times B \rightarrow B \times A$ such that f(a , b) = (b , a) is a bijective function.

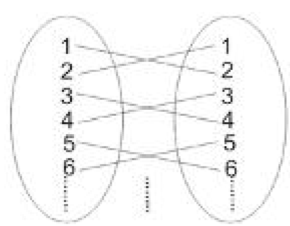
Sol.1) (Rough diagram)



One-One function
let $(a\,,b)$ and $(c\,,d)$ $\in$ $A \times B$(domain)
and $f(a\,,b$ $=$ f $(c\,,d)$
$\Rightarrow$ $(b\,,a)$ $=$ $(d\,,c)$
$\Rightarrow$ b $=$ d and a $=$ c
$\Rightarrow$ $(a\,,b)$ $=$ $(c\,,d)$
$\therefore$ f is one-one function
On-To
(.) since $n(A \times B) = n(B \times A)$
(.) f is one-one (just proved above)
(.) □ Range = co-domain
$\therefore$ f must be on o
$\therefore$ f is a bijective function ans.

Q.2) Show that $f: N \rightarrow N$ given by
$f(x)$ $=$ $\{x + 1$; $if\ x\ is\ odd\}$
$=$ $\{x - 1$; $if\ x\ is\ even\}$
f is a bijective function.

Sol.2) (Rough diagram)



One-One function :-
(.) Case 1 : let x_1, and x_2 on both odd
$x_1\,, x_2$ $\in$ N (domain)
and $f(x_1) = f(x_2)$
$\Rightarrow$ $x_1 + 1 = x_2 + 1$
$\Rightarrow$ $x_1 = x_2$
(.) Case 2: let x_1 , and x_2 both even
$x_1\,, x_2$ $\in$ N (domain)

and $f(x_1) = f(x_2)$

$\Rightarrow \quad x_1 - 1 = x_2 - 1$

$\Rightarrow \quad x_1 = x_2$

(.) Case 3 : let x_1 is odd and x_2 is even

and $f(x_1) = f(x_2)$

$\Rightarrow \quad x_1 + 1 = x_2 - 1$

$\Rightarrow \quad x_2 - x_1 = 2$ {not possible ∵ even no – odd no ≠ 2}

∴ thus case is rejected

(.) Case 4 : let x_1 is even and x_2 is odd

$f(x_1) = f(x_2)$

$\Rightarrow \quad x_1 - 1 = x_2 + 1$

$\Rightarrow \quad x_1 - x_2 = 2$ (not possible ∵ even no – odd no ≠ 2)

∴ thus case is also rejected

Hence, overall f is one-one function

On-To :

For every odd number $(2n - 1) \in N$ (co-domain) there exists an even number $(2n)$ in domain (N) and for every even number $(2p) \in N$ (co-domain) there exists an odd number $(2p - 1) \in N$(domain)

⇒ co-domain = Range

∴ f is on-to

∴ f is bijective function

Q.2) Given examples of two functions $f : N \to Z$ and $g : Z \to Z$ such that gof is injective but g is not injective.

Sol.2) Given : $f : N \to Z$

and $g : Z \to Z$

then domain of 'gof ' is same as domain of 'f ' and co-domain of 'gof ' is as same as co-domain of 'g'

∴ gof : N → Z

let $f(x) = x$ and $g(x) = |x|$

$gof = g(f(x))$

$= g(x)$

$gof = |x|$

one-one (for gof)

let $x_1, x_2 \in N$ (domain of gof)

and $(g0f)(x1) = (gof)(x_2)$

$\Rightarrow \quad g(f(x_1)) = g(f(x_2))$

$\Rightarrow \quad |x_1| = |x_2|$

$\Rightarrow \quad x_1 = \pm x_2$

but $x_1 \neq x_2$ ....$\{\because x_1, x_2 \in N\}$

$\therefore x_1 = x_2$

∴ gof is one-one function

Now $g(-1) = |-1| = 1$

$g(1) = |-1| = 1$

since two different elements in domain (z) of g has same image in co-domain (z)

∴ g is not one-one

$\therefore\ f(x) = x$ and $g(x) = |x|$ ans.

Q.3) (i) $f(x) = (3 - x^3)^{\frac{1}{3}}$. Find f0f(x)

Sol.3) (i) $f0f = f(f(x))$

$= f[(3 - x^3)^{\frac{1}{3}}]$

$= [3 - ((3 - x^3)^{\frac{1}{3}})^3]^{\frac{1}{3}}$

$= [3 - (3 - x^3)]^{\frac{1}{3}}$

$= [3 - 3 + x^3]^{\frac{1}{3}}$

$= (x^3)^{\frac{1}{3}}$

$= x$

$\therefore\ f0f = x$ ans.

(ii) $f(x) = |x|$

$g(x) = |5x - 2|$

Is fog = gof for all $x \in R$?

$fog = f(g(x))$

$= f(|5x - 2|)$

$= ||5x - 2||$

$= |5x - 2|$ $\{\ldots\ ||x|| = |x|\}$

$gof = g(f(x))$

$= g(|x|)$

$= |5|x| - 2|$

clearly $fog \neq gof$ ans.

e.g when $x = -1$

fog $= |5(-1) - 2| = |-5 - 2| = |-7| = 7$

$gof = |5|-1|-2| = |5 - 2| = 3$

(iii) If $f(x) = 2x$; $g(y) = 3y + 4$ and $h(z) = sin\ Z$

Show that ho(gof) = (hog)of

LHS $= ho\ (gof)$

$= ho\ [g(f(x))]$

$= ho\ [g(2x)]$

$= ho\ [3(2x) + 4]$

$= ho\ (6x + 4)$

$= sin\ (6x + 4)$

RHS $(hog)of$

$= [hog]of$

$= [h(g(y))]of$

$= [(3y + 4)]of$

$= [sin(3y + 4)]of$

$= sin(3y + 4)\ 0\ (2x)$

$= sin(3(2x) + 4) = sin\ (6x + 4)$

$\therefore$ LHS = RHS

Q.4) Let f : R → R be defined as $f(x) = 10x + 7$. Find function g(x) such that fog = gof = I_R

(where I_R = x identity function : R → real no's)

Sol.4) We have $f(x) = 10x + 7$

given $fog = gof = I_R$

$\Rightarrow fog = gof = x$ where $x \in R$

To find : g(x) :

Consider $fog = x$

$\Rightarrow f(g(x)) = x$

$\Rightarrow 10\, g(x) + 7 = x$

$\Rightarrow g(x) = \dfrac{x-7}{10}$

Now $gof = g(f(x))$

$= g(10x + 7)$

$= \dfrac{10x + 7 - 7}{10}$

$= \dfrac{10x}{10} = x = I_R \quad (verified)$

$\ldots\ g(x) = \frac{x-7}{10}$ ans.

Q.5) Let f = R → R be the sign um function defined as

$f(x) = \{-1\ ;\ x < 0\}$

$\{0\ ;\ x = 0\}$

$\{1\ ;\ x > 0\}$

and $g(x) = [x]$ be the greatest integer function. Then does fog and gof coincide (equal) in (0 , 1) ?

Sol.5) When $x \in (0, 1)$

value of $g(x) = [x]$ can be o or 1 ......{[0.1]=0} {[0.2]=0} {[1]=1}

value of $f(x) = 1$ ....{∵ when $x > 0\ f(x) = 1$}

Now $fog = f(g(x))$

$= f([x])$

$= f(0\ or\ 1)$ $\ldots\ldots\{as\ x \in [0, 1]\ [x]\ can\ be\ 0\ or\ 1\}$

$= 0, 1$ ......{when $x = 0\ ;\ f(x) = 0$, when $x = 1\ ;\ f(x) = 1$}

Now $gof = g(f(x))$

$= g(1)$ $\ldots\ldots\{\ldots x \in [0, 1]\ ,\quad f(x) = 1\}$

$= [1] = 1$

clearly fog does not coincide (equal) with gof when $x \in [0, 1]$ ans.

Q.6) Let f : {1 ,3,4} → {1,2,5} and g : {1,2,5} → {1,3} be given by f = {(1,2), (3,5), (4,1)} and g = {(1,3), (2,3), (5,1)} write down gof.

Sol.6) Domain of gof is same as domain of f and co-domain of gof is same as co-domain of g

∴ gof : {1,3,4} → {1,3}

Now , given : f(1) = 2 g(1) = 3

f(3) = 5 g(2) = 3

f(4) = 1 g(5) = 1

gof(1) = g(f(1)) = g(2) = 3

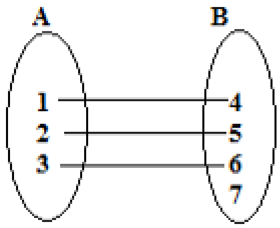
gof(3) = g(f(3)) = g(5) = 1

gof(4) = g(f(4)) = g(1) = 3

∴ gof = {(1 , 3), (3 , 1), (4 , 3)} ans.

Q.7) Let A = {1,2,3} and B = {4,5,6,7} and f = {(1 , 4), (2 , 5), (3 , 6)} be a function from A to B. State whether f is one-one or on-to.

Sol.7) Given f(1) = 4 f(2) = 5 f(3) = 6



Clearly f is one-one , as every element in domain (A) has a unique image in co-domain (B)
Since 7 ∈ co-domain (B), but this is not the image of any element in domain (A)
∴ f is not on-to ans.

Q.8) Consider the function $f:\left[0,\frac{\pi}{2}\right] \rightarrow R$given by $f(x) = \sin x$and $g:\left[0,\frac{\pi}{2}\right] \rightarrow R$given by g(x) = cos x. Show that f and g are one-one but f + g is not one-one.

Sol.8) We know that for any two different elements $x_1$and $x_2 \in \left[0,\frac{\pi}{2}\right]$

$\sin x_1 \neq \sin x_2 \text{and} \cos x_1 \neq \cos x_2$

□ $f(x_1) \neq (f)x_2 \text{and} g(x_1) \neq g(x_2)$

for all x_1 , x_2 □ [0 , 7] and $x_1 \neq x_2$

∴ f and g are one-one

Now $f + g = \sin x + \cos x$

$(f+g)(0) = f(0) + g(0) = \sin(0) + \cos(0) = 0 + 1 = 1$

$(f+g)\left(\frac{\pi}{2}\right) = f\left(\frac{\pi}{2}\right) + g\left(\frac{\pi}{2}\right) = \sin\left(\frac{\pi}{2}\right) + \cos\frac{\pi}{2} = 1 + 0 = 1$

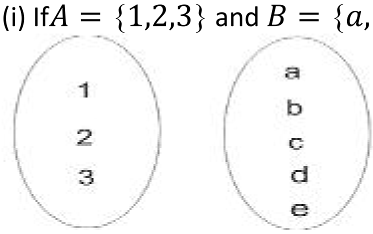
clearly $(f+g)(0) = (f+g)\left(\frac{\pi}{2}\right)$

but $0 \neq \frac{\pi}{2}$

i.e two different elements in domain $\left[o,\frac{\pi}{2}\right]$has same image in co-domain (R)

∴ $f + g$ is not one-one ans.

Q.9) (i) If$A = \{1,2,3\}$ and $B = \{a,c,d,e\}$. Find number of one-one functions

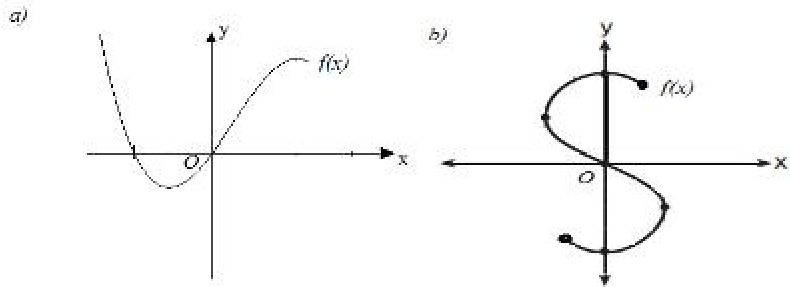


(ii) Find the number of on-to function from A to A if A = {1,2,3......n}

Sol.9) (i) The element 1 in A can be attached / associated with any element of B in 4 ways
element 2 in A can be attached / Associated in 3 ways
and element 3 can be associated in 2 ways
∴ total no. of one-one function = $4 \times 3 \times 2 = 24$ ans.

(ii) The element 1 in co-domain can be attached / Associated with any element of domain in = n

ways
element 2 can be associated in = $(n - 1)$ ways
element 3 can be associated in = $(n - 2)$ ways
element n can be associated in = 1 way
$\therefore$ the total no of on-to function an = $n \times (n-1) \times (n-2) \times \ldots..1 = n!$ans.

Q.10) 

Which of the following graphs represent a function ?

Sol.10) (a) is a function
$\because$ for each value of x , $f(x)$ attains a unique and different value.
(b) is not a function
since for same value of x , f(x) has multiple values.