# Class 12th
## Relations & Functions

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| Q.1) | Show that the relation $R$ in set $Z$ given by $R\{(a\,,b)\,:\,2\;divides\;\;a-b\}$ is an Equivalence relation. |
| Sol.1) | We have, $R\;=\;\{(a\,,b)\,:\,2\;divide\;a-b\}$<br>Symmetric :<br>let $(a\,,b)\;\in\;R$<br>$\Rightarrow\;a-b\;is\;divisible\;by\;2$<br>$\Rightarrow\;a-b\;=2\lambda$ ......$\{\lambda\epsilon Z\}$<br>$\Rightarrow\;b-a\;=-2\lambda$ which is also divisible by 2<br>$\Rightarrow\;(b\,,a)\;\in$<br>$\therefore$ R is Symmetric<br>Reflexive :<br>for each $a\;\in\;Z$<br>$\Rightarrow\;a-a\;=\;0$ which is divisible by 2<br>$\Rightarrow\;(a\,,a)\;\in\;R$<br>$\therefore$ R is Reflexive<br>Transitive :<br>let $(a,b)\;\in\;R$ and $(b\,,c)\;\in\;R$<br>$\Rightarrow\;a-b\;=2\lambda$ and $b-c\;=\;2\;k$ .....$\{\lambda,k\epsilon Z\}$<br>Now , $a-c\;=\;(a-b)\;+\;(b-c)$<br>$\Rightarrow\;a-c\;=2\lambda+2k$<br>$\Rightarrow\;a-c\;=2(\lambda+k)$which is also divisible by 2<br>$\Rightarrow\;(a\,,c)\;\in\;R$<br>$\therefore$ R is transitive<br>since $R$ is Symmetric , Reflexive and transitive<br>$\therefore R$ is an Equivalence relation ans. |
| Q.2) | Show that the relation R in the set A, $A\;=\;\{x\;\in\;z\;:\;0\;\leq\;x\;\leq\;12\}$ given by $R\;=\;\{(a,b)\,:\,(a-b)$ is multiple of 4} is an equivalence relation. Find the set of all the elements in set A which are related to 1. |
| Sol.2) | We have , $R\;=\;\{(a\,,b)\,:\,\lvert a-b\rvert$ is multiple of 4}<br>Symmetric :<br>let $(a,b)\;\in\;R$<br>$\Rightarrow\;\lvert a-b\rvert$ is multiple of 4<br>$\Rightarrow\;\lvert a-b\rvert\;=4\lambda$ ......$(\lambda\epsilon z)$<br>$\Rightarrow\;\lvert b-a\rvert\;=4\lambda$ which is multiple by 4<br>$\Rightarrow\;(b,a)\;\in\;R$<br>$\therefore$ R is Symmetric<br>Reflexive :<br>for each $a\;\in\;A$<br>we have, $\lvert a-a\rvert\;=\;0$ which is multiple of 4<br>$\Rightarrow\;(a\,,a)\;\in\;R$<br>$\therefore$ R is Reflexive<br>Transitive : |

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| | let $(a\,,b)\;\in\;R$ and $(b\,,c)\;\in\;R$<br>$\Rightarrow\;\lvert a - b\rvert\;=4\lambda$ and $\lvert b - c\rvert\;=\;4k$ .....$\{\lambda, k\epsilon Z\}$<br>$\Rightarrow\;(a - b)\;=\pm 4\lambda$ and $(b - c)\;=-4k$<br>Now , $(a - c)\;=\;(a - b)\;+\;(b - c)$<br>$\Rightarrow\;(a - c)\;=\pm 4\lambda \pm 4k$<br>$\Rightarrow\;(a - c)\;=\pm 4(\lambda + k)$<br>$\Rightarrow\;\lvert a - c\rvert\;=\;\lvert \lambda\;+\;k\rvert$ which is multiple of 4<br>$\Rightarrow\;(a\,,c)\;\in\;R$<br>$\therefore$ R is transitive<br>$\therefore$ R is an Equivalence relation<br>The elements which related to 1 are 1, 5, 9<br>$\therefore$ required set is {1, 5, 9} ans. |
| Q.3) | Let R be a relation on the set "A" of ordered pairs defined by $(x\,,y)\;R(u\,,v)$ if and only if $xv\;=\;yu.$ Show that R is an equivalence relation. |
| Sol.3) | Given : A → set of ordered pairs<br>$(x\,,y)\;R(u\,,v) \Rightarrow\;xv\;=\;yu$<br>Symmetric :<br>let $(x\,,y)\;R\;(u\,,v)$<br>$\Rightarrow\;xv\;=\;yu$ \| (Rough work)<br>$\Rightarrow\;vx\;=\;uy$ \| $((u\,,v)\;R(x\,,y))$<br>$\Rightarrow\;uy\;=\;vx\;\;\Rightarrow\;\;(4\,,v)\;R(x\,,y)$ \| $(uy\;=\;vx)$<br>$\therefore$ R is Symmetric<br><br>Reflexive :<br>for each (x , y) ∈ A \| {Rough work}<br>$\Rightarrow\;xy\;=\;yx$ \| $\{(x\,,y)\;R(xy)\}$<br>$\Rightarrow (x\,,y)\;R(x\,,y)$ \| $\{xy\;=\;yx\}$<br>$\therefore$ R is Reflexive<br><br>Transitive :<br>let $(x\,,y)\;\;R(u\,,v)$ and $(u\,,v)\;\;R(a\,,b)$<br>$\Rightarrow\;xv\;=\;yu$ and $ub\;=\;va$<br>$\Rightarrow\;xv\;=\;yu$ and $v = \frac{ub}{a}$ $\ldots..\{Rough\;(x\,,y)\;R(a\,,b)\;\;,\;xb\;=\;ya\}$<br>$\Rightarrow\;x\left(\frac{ub}{a}\right)$= yu<br>$\Rightarrow\;xb\;=\;ya$<br>$\Rightarrow\;(x\,,y)\;R(a\,,b)$<br>$\therefore$ R is transitive<br>since R is Symmetric , Reflexive as well as transitive<br>$\therefore$ R is an Equivalence relation ans. |
| Q.4) | If $R_1$ and $R_2$ are equivalence relations in set A , show that $R_1 \cap R_2$is also on equivalence relation. |
| Sol.4) | Given :- $R_1$ and $R_2$ are equivalence relations<br>Symmetric : |

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| | let $(a\,,b) \in R_1 \cap R_2$<br>$\Rightarrow (a\,,b) \in R_1$ and $(a\,,b) \in R_2$<br>$\Rightarrow (b\,,a) \in R_1$ and $(b\,,a) \in R_2$ ....{∵ R and R are symmetric relations}<br>$\Rightarrow (b\,,a) \in R_1 \cap R_2$<br>∴ $R_1 \cap R_2$is Symmetric<br>Reflexive :<br>for each $a \in A$<br>we have, $(a\,,a) \in R_1$ and $(a\,,a) \in R_2$ $\ldots\ldots\ldots\ldots.\{R1\ and\ R_2\ are\ reflexive\}$<br>$\Rightarrow (a\,,a) \in R_1 \cap R_2$<br>∴ $R_1 \cap R_2$is Reflexive<br>Transitive :<br>let $(a\,,b) \in R_1 \cap R_2$ and $(b\,,c)R_1 \cap R_2$<br>$\Rightarrow (a\,,b) \in R_1\ and\ (a\,,b) \in R_2\ \ and\ (b\,,c) \in R_1\ \&\ (b\,,c) \in R_2$<br>$\Rightarrow (a\,,b) \in R_1\ and\ (b\,,c) \in R_1$ $\mid (a\,,b)\ R\ and\ (b\,,c) \in R_2$<br>$\Rightarrow (a\,,c) \in R$ $\mid (a\,,c) \in R_2$<br>$\ldots.\{R_1\ \&\ R_2\ are\ transitive\}$<br>$\Rightarrow (a\,,c) \in R_1 \cap R_2$<br>∴ $R_1 \cap R_2$is transitive<br>since $R_1 \cap R_2$ is Symmetric , Reflexive as well as transitive<br>∴ $R_1 \cap R_2$is an Equivalence relation ans. |
| Q.5) | $R$ is a relation on set $N$ given by $aRb \leftrightarrow b$ is divisible by $a;\ a.\,b \in N$check whether R is Symmetric , reflexive and transitive. |
| Sol.5) | We have,$aRb \leftrightarrow b$ is divisible by $a$<br>Symmetric :<br>$2R6 \Rightarrow 6$ is divisible by 2 ....$\left\{\frac{6}{2} = 3\right\}$<br>but $6R2 \Rightarrow 2$is not div by 6 .....$\left\{\frac{2}{6} = \frac{1}{2}\right\}$<br>∴ R is not symmetric<br>Reflexive : for each $a \in N$<br>a is always divisible by a<br>$\Rightarrow aRa$<br>∴ R is Reflexive<br>Transitive :<br>let $aRb$ and $bRc$<br>$\Rightarrow$ b is divisible by a and c is div by b<br>$\Rightarrow$ $b = a\lambda$ and $c = bk$ ....$\{\lambda, k \epsilon N\}$<br>$\Rightarrow$ $c = (a\lambda)k$ ....$\{\because b = a\lambda\}$<br>$\Rightarrow \frac{c}{a} = \lambda k$<br>clearly c is div by a<br>$\Rightarrow aRc$<br>∴ R is transitive ans. |
| Q.6) | R be relation in P(x) , where x is a non-empty set , given by<br>ARB if only if ACB , where A & B are subsets in $P(x)$. Is R is an equivalence relation on $P(x)$ ? Justify |

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| | your answer. |
| Sol.6) | Let ARB<br>$\Rightarrow A \subset B$<br>then it is not necessary that B is a subset of A<br>i.e. $B \not\subset A$<br>⇒ B R A<br>∴ R is not symmetric and hence R is not an equivalence relation<br>eg. $x = \{1,2,3\}$<br>$P(x) = \{\{1\}\{2\}\{3\}\{1,2\}\{2,3\}\{1,3\}\{1,2,3\}\}$<br>clearly $\{2\} \subset \{1,2\}$<br>between$\{1,2\} \subset \{2\}$<br>∴ R is not symmetric ans. |
| Q.7) | Show that the relation R defined in the set A of all triangles as R-{($T_1$ , $T_2$) : $T_1$ is similar to $T_2$} is equivalence relation.<br>Consider three right angle triangles $T_1$ with sides 3, 4, 5, $T_2$ with sides 5, 12, 13 and $T_3$ with sides 6, 8, 10. Which triangles among $T_1$ , $T_2$ and $T_3$ are related ? |
| Sol.7) | A → set of are triangles<br>$R = \{(T_1, T_2) : T_1 \sim T_2\}$<br>Symmetric :<br>let $(T_1, T_2) \in R$<br>$\Rightarrow T_1 \sim T_2$<br>$\Rightarrow T_2 \sim T_1$<br>$\Rightarrow (T_2, T_1) \in R$<br>∴ R is symmetric<br><br>Reflexive : for each triangle $T \in A$<br>$(T, T) \in R$<br>since every triangle is similar to itself<br>∴ R is reflexive<br>Transitive :<br>let $(T_1, T_2) \in R$ and $(T_2 \sim T_3) \in R$<br>$\Rightarrow T_1 \sim T_2$ $and\ T_2 \sim T_3$<br>$\Rightarrow T_1 \sim T_3$<br>$\Rightarrow T_1, T_3) \in R$<br>∴ R is transitive<br>and hence R is an equivalence relation<br>$T_1 : 3, 4, 5$<br>$T_2 : 5, 12, 13$<br>$T_3 : 6, 8, 10$<br>clearly sides of triangles $T_1$ and $T_3$ are in equal proportion i.e $\frac{3}{6} = \frac{4}{8} = \frac{5}{10}$<br>∴ $T_1 \sim T_3$<br>⇒ $T_1$ and $T_3$ are related to each other ans. |
| Q.8) | Check whether the relation R in R (real no's) define by $R = (a, b): a \leq b^3$is reflexive, symmetric or transitive. |

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| Sol.8) | Symmetric :<br>$(1,2) \in R$<br>as$1 \leq 2^3$<br>but $(2,1) \notin R$<br>since$2 \nleq 13$<br>∴ R is not symmetric<br>Reflexive : $\frac{1}{2} \in R$<br>but $\left(\frac{1}{2},\frac{1}{2}\right) \notin R$<br>as $\frac{1}{2} \nleq \left(\frac{1}{2}\right)^3$<br>∴ R is not reflexive<br>Transitive :<br>$(9,4) \in R$and$(4,2) \in R$<br>as $9 \leq 4^3$ and $4 \leq 2^3$<br>but$(9,2) \notin R$<br>since $9 \nleq 2^3$<br>∴ R is not transitive ans. |
| Q.9) | Show that the relation R in the set {1,2,3} given by R = {(1 , 1), (2 , 2), (3 , 3), (1 , 2), (2 , 3)} is reflexive neither symmetric nor transitive. |
| Sol.9) | We have,<br>$A = \{1,2,3\}$<br>$R = \{(1,1),(2,2),(3,3),(1,2),(2,3)\}$<br>since $(1,2) \in R$<br>but $(2,1) \notin R$<br>∴ R is not Symmetric<br>$(1,2) \in R$and$(2,3) \in R$<br>but $(1,3) \notin R$<br>∴ R is not transitive<br>for each $a \in A$<br>$(a\,,a) \in R$ i.e. $(1,1),(2,2),(3,3) \in R$<br>∴ R is reflexive ans. |
| Q.10) | Determine whether each of the following relations are reflexive, symmetric and transitive<br>(i) Relation in set A = {1,2,3,.......... 13,14} defined by $R = (x,y)\colon 3\text{x}- y = 0$.<br>(ii) Relation in N defined as $R = (x,y)\colon y = x + 5; x < 4$.<br>(iii) Relation in set A = {1,2,3,4,5,6} defined as $R = (x,y)\colon y$is divisible by $x$.<br>(iv) Relation in Z defined as $R = (x,y)\colon x- y$is an integer.<br>(v) Relation in R (real nos) defined as $R = (a,b)\colon a \leq b^2$. |
| Sol.10) | (i) $R = \{(1,3),(2,6),(3,9),(4,12)\}$ ......$(y = 3\text{x})$<br>clearly $(1,3) \in R$but$(3,1) \notin R$<br>∴ not symmetric<br>$1 \in A$ but $(1,1) \notin R$<br>∴ not reflexive |

$(1,3) \in R$and $(3,9) \in R$but$(1,9) \notin R$
$\therefore$ not transitive

(ii) $R = \{(1,6),(2,7),(3,8)\}$ $\ldots..\{\ldots\ y = x + 5\ and\ x < 4\}$
Do yourself

(iii) $R = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,2),(2,4),(2,6),(3,3),(3,6),(4,4),(5,5),(6,6)\}\ldots\{\ldots y\ is\ divisible\ by\ x\}$

clearly for each $a \in A$
$(a, a) \in R$ i.e. $(1,1),(2,2),(3,3),(4,4),(5,5),(6,6) \in R$
$\therefore$ R is reflexive
$(1, 2) \in R$
but$(2,1) \notin R$
since 1 in not divisible by 2
$\therefore$ R is not transitive
for each $(a, b)$ and $(b, c) \in R$
clearly $(a, c) \in R$
$\therefore$ R is transitive

(iv) Symmetric let $(x, y) \in R$
$\Rightarrow$ $x - y = \lambda$ ..... where $\lambda \rightarrow$ integer
$\Rightarrow$ $y - x = -\lambda$ which is also an integer
$\Rightarrow (y, x) \in R$
$\therefore$ R is Symmetric
Reflexive and transitive (Do yourself)

(v) give same examples as in case of $a \leq b^3$
It is neither symmetric, nor reflexive, nor transitive.