RELATIONS AND FUNCTIONS

FOUR MARKS QUESTIONS

1. Show that the relation R in the set N of Natural numbers given by
R = $\{(a, b): |a - b| \text{ is a multiple of } 3\}$ is an equivalence relation.
Determine whether each of the following relations are reflexive, symmetric, and Transitive.

2. Check whether the relation R in R defined by R = $\{(a, b): a \leq b^3\}$ is reflexive, symmetric, transitive.

3. Prove the relation R on the set N x N defined by (a, b) R (c, d) ⇔a+d = b + c, for all (a, b) (c, d) є N x N is an equivalence relation.

4. Prove that the function *f:* **R**⟶ **R,** given by *f (x)* = $|x|$ + 5, is not bijective.

5. Prove that the function *f:* **R**⟶ **R,** given by *f (x)* =4x^3 -7, is bijective

6. Prove that the Greatest Integer Function f: R⟶**R** given by f(x) =$[x]$, is neither one-one nor onto where $[x]$ denotes the greatest intger less than or equal to x .

7. Let *f:* **N** →**N** be defined by

$$f(n) = \begin{cases} \frac{n+1}{2} \text{ if } n \text{ is odd} \\ \frac{n}{2} \text{ if } n \text{ is even} \end{cases} \text{ for all } n \quad \mathbf{N},$$

State whether the function f is bijective.

9. Consider that f: **N** →N given by f(x) = $x^2 + x + 1$. Show that f is not invertible.

10. Let A=**N x N** and * be the binary operation on **A** defined by
(a, b) *(c, d) = (ad + bc, bd) .Show that * is commutative and associative . Find the identity element for * on A , if any.

11. Let f: R→R be the function defined by $f(x) = \frac{1}{2 - cosx} \forall x \in R$.Then find the range of *f.* (Exemplar).

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11. Let f: **N**→ $\boldsymbol{R}$ be a function defined as f(x) = 4x^2 +12x +15. Show that f:**N**→ $\boldsymbol{S}$, where **S** is the range of f is invertible .Find the inverse of f.

12. Consider f: **R**+→ $|[-5, \ )$given by f(x) =9$x^2 + 6x - 5$. Show that f is invertible. Find the inverse of f.

13. Let * be the binary operation on Z given by a*b = a + b-15. 1) Is * commutative?
2) Is * associative 3) Does the identity for *exist? If yes find the identity.
4) Are the elements of Z invertible? If so find the inverse.

14.Let A = $A = R - \{3\}, B = R - \{1\}. Let\ f: A \to B\ defined\ by\ f(x) = \frac{x-2}{x-3} \forall x \in A.$
Then show that f is bijective.(Exemplar).