Linear Programming (LPP)

Class 12^{th}

Q.1) A merchant plans to sell two types of personal computer a desktop model and a portable model that will costs Rs 25000 and Rs 40,000 respectively. He estimates that the total monthly demand of computers will not exceed 250units. Determine the number of units of each type of computers which the merchant should stock to get maximum profit if he does not want to invest more than Rs 70 lakh and if his profit on the desktop model is Rs 4500 and on portable model is Rs 5000.

Sol.1) Let the merchant stock x desktop models and y portable models.

Therefore, $x \geq 0$ and $y \geq 0$

The cost of a desktop model is Rs 25000 and of a portable model is Rs 4000. However, the merchant can invest a maximum of Rs.70 lakhs.

$25000x + 40000y \leq 7000000$

$5x + 8y \leq 1400$

The monthly demand of computers will not exceed 250 units.

$\therefore x + y \leq 250$

The profit on a desktop model is Rs. 4500 and the profit on a portable model is Rs. 5000.

Total profit, $Z = 4500x + 5000y$

Thus, the mathematical formulation of the given problem is

Maximum $Z = 4500x + 5000y$ ………….. (i)

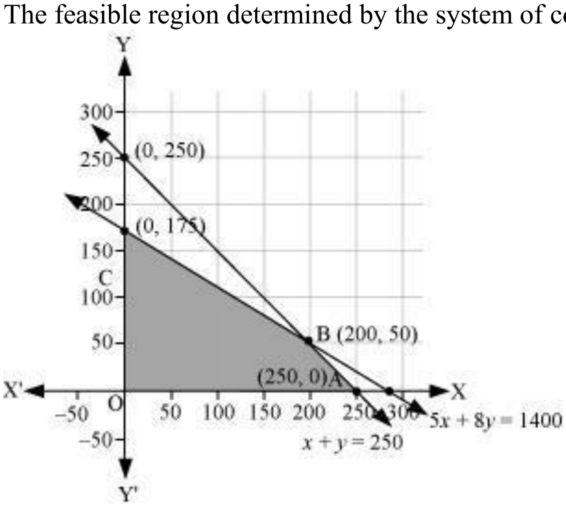
subject to the constraints,

$5x + 8y \leq 1400$ ………. (ii)

$x + y \leq 250$ ………. (iii)

$x, y \geq 0$ ………. (iv)

The feasible region determined by the system of constraints is as follows:



The corner points are A (250, 0), B (200, 50), and C (0, 175).

The values of Z at these corner points are as follows.

Corner point	$Z = 4500x + 5000y$

A(250, 0)	1125000
B(200, 50)	1150000 → Maximum
C(0, 175)	875000

The maximum value of Z is 1150000 at (200, 50).
Thus, the merchant should stock 200 desktop models and 50 portable models to get the maximum profit of Rs.1150000 ans.

Q.2) Reshma wishes to mix two types of food pond Q in such a way that the vitamin contents of the mixture contain at least 8 $units$ of vitamin A and II units of vitamin B. Food P costs $Rs. 60 / kg$ and food Q costs $Rs. 80 / kg$. Food P contains 3 $units / kg$ of vitamin A and 5 $units / kg$ of vitamin B while food Q contains 4 $units / kg$ of vitamin A and 2 $units / kg$ of vitamin B. Determine the minimum cost of the mixture.

Sol.2) Let the mixture contain $x\ kg$ of food P
& $y\ kg$ of food Q

Resources	**Food**		**Requirement**
	P	**Q**	
Vitamin A	3	4	At least 5 units
Vitamin B	5	2	At least 11 units
Cost	60	80	

According to question:
Vitamin A contains :-
Food P contains : 3 units
Food Q contains : 4 units
Total requirements : 8 units
$\therefore 3x + 4y \geq 8$
$x \geq 0, y \geq 0$

Vitamin B contains :-
Food P contains : 5 units
Food Q contains : 2 units
Total requirements : 11 units
$\therefore 5x + 2y \geq 11$
$x \geq 0, y \geq 0$

As we need to minimize cost of mixture, the function formed in minimize Z
Cost of food P per kg = Rs.60
Cost of food Q per kg = Rs.80
∴ minimum cost of mixture :
Minimize Z $= 60x + 80y$

Hence, combining the constraints:
Minimize Z $= 60x + 80y$
Subject to constraints
$3x + 4y \geq 8$
$5x + 2y \geq 11$

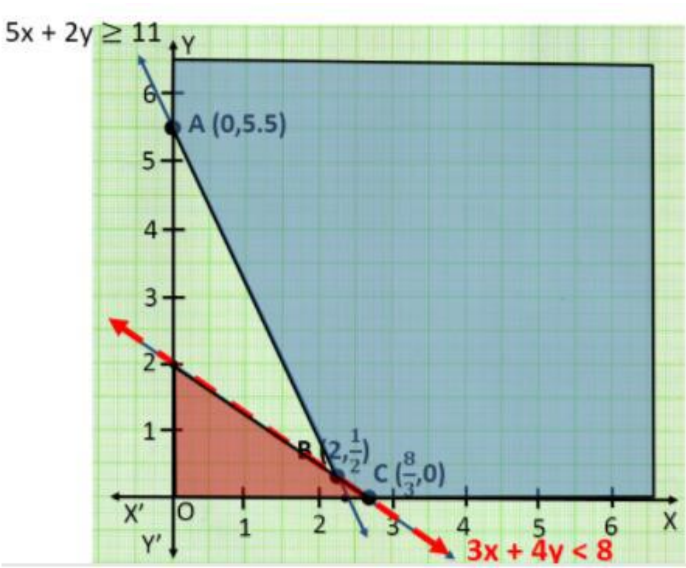
$x \geq 0, y \geq 0$

$3x + 4y \geq 8$
$x = 0, \frac{8}{3}$ & $y = 2, 0$

$5x + 2y \geq 11$
$$x = 0, \frac{11}{5}$$
$$y = \frac{11}{2}, 0$$

Corner points	Value of Z	
(0, 5.5)	440	
(2, ½)	160	Minimum
(8/3, 0)	160	

Since the feasible region is unbounded,
Hence 380 may or may not be the minimum value of Z
So, we need to graph inequality:
$60x + 80y < 160$
$3x + 4y < 8$



As, there is no common points between the feasible region inequality.
∴ 160 is the minimum value of z
Since, Z is minimum at two points (8/3 , 0) & (2, ½)
Hence, the minimum cost of mixture is Rs.160 at all joining points (8/3 , 0) & (2, ½)

Q.3) A man owns a field of area 1000 $sq.m.$ He wants to plant fruit trees in it. He has a sum of Rs 1400 to purchase young trees. He has the choice of two types of trees. Type A requires 10 sq. m of ground per tree and costs Rs 20 per tree and type B requires 20 sq m of ground per tree and costs Rs. 25 per tree. When fully grown, type A producer an average of 20 kg of fruit which can be sold at a profit of Rs. 2 per kg and type B producer an average of 40 kg of fruit which can be sold at a profit of Rs 1.50 per kg. How many of each type should be planned to achieve maximum profit? What is the maximum profit ?

Sol.3) Let x trees of type A and y trees of type B are planted

let Z be the total profit

Total cost for purchasing trees = $20x + 25y$

Total revenue $=(20 \times 2)x + (40 \times 1.5)y$

$= 40x + 60y$

$\therefore$ total profit = total revenue – total profit

$= (40x + 60y) - (20x + 25y)$

$= 20x + 35y$

LPP :

Maximize $z = 20x + 35y$

subject to constraints

$20x + 25y \leq 1400$ …..(investment constraints)

$10x + 20y \leq 1000$ …..(area available constraint)

and $x, y \geq 0$

Type A : 20 trees

Type B : 40 trees and Max profit = Rs. 2200

Q.4) A manufacturing company makes two types of teaching aids A and B of mathematics for class XII. Each type of A require 9 labour hours of fabricating & 1 labour hour for finishing. Each type B requires 12 labour hours for fabricating & 3 labour hrs of finishing. For fabrication & finishing, the maximum labour hours per week are 180 & 30 respectively. The makes a profit of Rs. 80 on each piece of type A & Rs.120 on each piece of type B. How many pieces of type A & type B should be manufactured per week to get a maximum profit? Make it as an LPP & solve graphically. What is the maximum profit per week?

Sol.4) Let x & y be the number of pieces of type A & type B manufactured per week respectively. If Z be the profit then,

Objective function, Z $=80x + 120y$

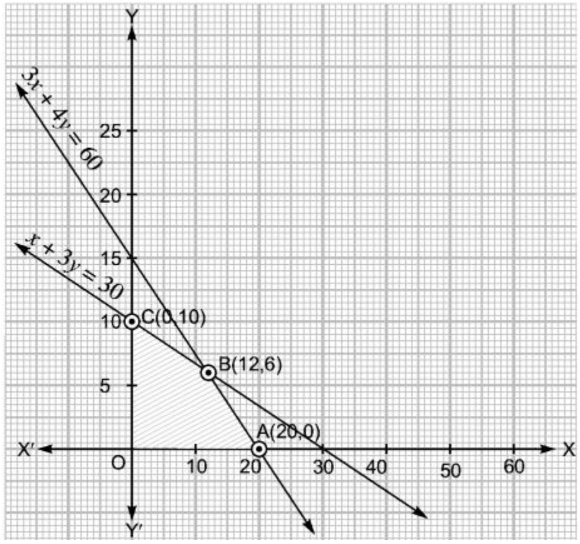
We have to maximize Z, subject to the constraints

$9x + 12y \leq 180 \Rightarrow 3x + 4y \leq 60$ ……. (i)

$x + 3y \leq 30$ …….. (ii)

$x \geq 0, y \geq 0$ …….. (iii)

The graph of constraints are drawn & feasible region OABC is obtained, which is bounded having corner points O(0,0), A(20, 0), B(12,6), C(0, 10)



Now the value of objective function is obtained at corner points as:

Corner point	$Z = 80x + 120y$
O(0, 0)	0
A(20, 0)	1600
B(12, 6)	1680 (maximum)
C(0, 10)	1200

Hence, company will get the maximum profit of Rs.1680 by making 12 pieces of type A & 6 pieces of type B of teaching aid.

Q.5) A diet for a sick person must contain at least 4000 units of vitamins 50 units of mineral and 1400 of calories. Two foods B and A are available at cost of Rs 3 and Rs 4 per unit respectively. If one unit of A contains 200 units of vitamin, 1 unit of mineral and 40 calories one unit of B contains 2 units of minerals 100 units of vitamins and 40 calories. Find what combination of food should be used to have the least cost ?

Sol.5) Suppose the diet contains x $units$ of food A & y $units$ of food B. since, one unit of food A costs Rs.4 & one unit of food B costs Rs.3.

Therefore, total cost of x $units$ of food A & y $units$ of food B is Rs.$(4x + 3y)$

Let Z denotes the total cost

$Z = 4x + 3y$

Since each unit of food A contains 200 $units$ of vitamins. Therefore, x $units$ of food A contain $200x$ $units$ of vitamins. A unit of food B contains 100 $units$ of vitamins. Therefore, y $units$ of food B contains 100 y $units$ of vitamins.

Thus, x $units$ of food A & y $units$ of food B contain $200x + 100y$ $units$ of vitamins. But the minimum requirements of vitamins is 4000 $units$.

$\therefore 200x + 100y \geq 4000$

Similarly, the total amount of minerals supplied by x $units$ of food A & y $units$ of food B is $x + 2y$ & the minimum requirements is of 50 $units$.

$\therefore x + 2y \geq 50$

Finally, the total calories in x $units$ of food A and y $units$ of food B is $40x + 40y$ and the minimum Calories required is 1400 units.

$\therefore 40x + 40y \geq 1400$

Clearly, $x \geq 0$ and $y \geq 0$

Since, we have to minimize e the total cost Z. Thus, the mathematical form of the given LPP is as follows:

Minimize $Z = 4x + 3y$

Subject to

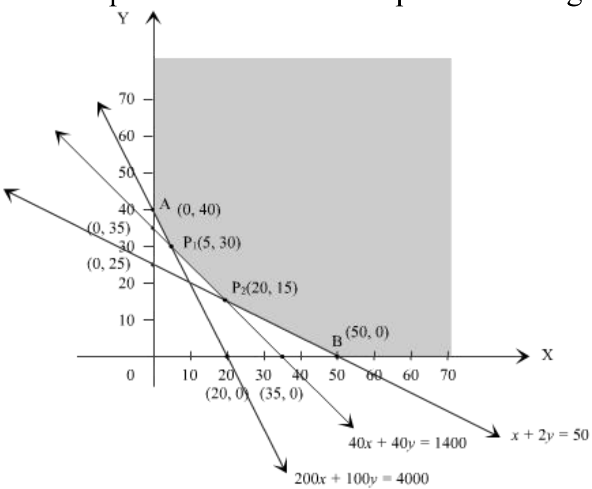
$200x + 100y \geq 4000$

$x + 2y \geq 50$

$40x + 40y \geq 1400$

$x, y \geq 0$

The inequalities can now be represented on graph as:



The solution set of the linear constraints is shaded in the figure. The vertices of the shaded region A(0,40), $P_1(5,30), P_2(20,15)$ and B (50, 0).

The value of the objective function at these points are given in the following table.

Point (x, y)	$\boldsymbol{Z = 4x + 3y}$
A(0,40)	120
$P_1(5,30)$	110
$P_2(20,15)$	125
B(50,0)	200

Clearly, Z is minimum for $x = 5$ and $y = 30$ and the minimum value of Z is 110.

Hence, the diet cost is minimum when 5 units of food A and 30 units of food B are taken.

The minimum diet cost is Rs.110.

Q.6) A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type A require 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B require 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours 20 minutes available for cutting and 4 hours of assembling. The profit is Rs 5 each for type A and Rs 6 each for type B souvenirs. How many souvenirs of each type should the company manufacture in order to maximize the profit?

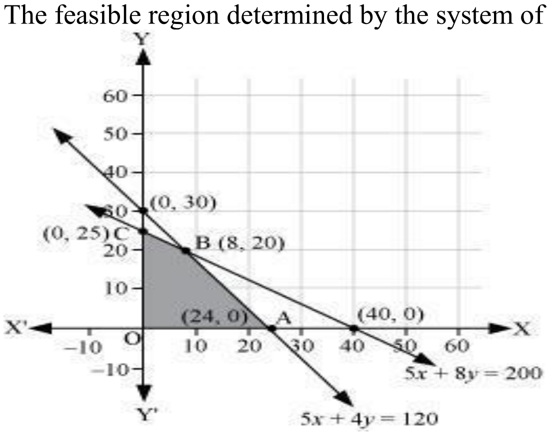
Sol.6) Let, the company manufacture x souvenirs of type A and y souvenirs of type B.

Therefore, $x \geq 0$ and $y \geq 0$

The given information can be compiled in a table as follows:

	Type A	Type B	Availability
Cutting (min)	5	8	$3 \times 60 + 20 = 200$
Assembling (min)	10	8	$4 \times 60 = 240$

The profit on type A souvenirs is Rs 5 and on type B souvenirs is Rs 6.
Therefore, the constraints are:
$5x + 8y \leq 200$
$10x + 8y \leq 240$ $i.e., 5x + 4y \leq 120$
Total profit, $Z = 5x + 6y$
The mathematical formulation of the given problem is
Maximize $Z = 5x + 6y$ ….. (i)
subject to the constraints,
$5x + 8y \leq 200$ ….. (ii)
$5x + 4y \leq 120$ ….. (iii)
$x, y \geq 0$ …. (iv)
The feasible region determined by the system of constraints is as follows:



The corner points are A (24, 0), B (8, 20), and C (0, 25).
The values of Z at these corner points are as follows:

Corner point	$Z = 5x + 6y$	
A (24, 0)	120	
B (8, 20)	160	⇒ Maximum
C (0, 25)	150	

The maximum value of Z is 200 at (8, 20).
Thus, 8 souvenirs of type A and 20 souvenirs of type B should be produced each day to get the maximum profit of Rs. 160.