Linear Programming (LPP)

Class 12th

Q.1) A small manufactured has employed 5 skilled me n & 10 semi-skilled men and makes an article in two qualities deluxe model and an ordinary model. The making of a deluxe model requires $2\ hrs$ work by skilled man and $2\ hr$ work by a semi-skilled man. The ordinary model requires $1\ hr$ by a skilled man and $3hrs$ by a semi-skilled man. By union rules no man may work more than $8\ hrs$ per day. The profit on a deluxe model is $Rs\ 15$ and an ordinary model is $Rs\ 10$. How many of each type should be made to maximize his daily profit ?

Sol.1) Let the company produce x deluxe model & y ordinary model article
Hence, total time spent by 5 skilled men $= 2x + y$ & it should be less than
Hence, $2x + y =\leq 40$ ………… (i)

Also, the total time spent by 10 semi-skilled men $= 2x + 3y$ & it should be less than 8 hrs / person
Hence, $2x + 3y =\leq 80$ ………… (ii)

Also, $x \geq\ 0$ ………….. (iii)
And $y \geq 0$ ………….. (iv)

Profit $= 15x + 10y$
We need to maximize profit given constraints. So, for that draw all the 4 lines on the graph & check the common area, you will form a hexagon $OABC$ with A(0, 80/3) , B(10 , 20) , C(20, 0) , O(0 , 0)

Since the feasible region is a bound region, we can check the profit function at all the vertices to find the maxima
At point A: 15(0) + 10(80/3) = 800/3
At point B: 15(10) + 10(20) = 350
At point C: 15(20) + 10(10) = 300
At point O: 15(0) + 10(0) = 0

So, the maxima lies at point C with $x = 10$ & $y = 20$ and the maximum profit $= Rs. 350$
10 deluxe models & 20 ordinary models max. profit Rs.350 ans.

Q.2) An aeroplane can carry a maximum of 200 passengers. A profit of Rs.1000 is made on each executive class tickets and a profit of Rs. 600 is made on each economy class tickets. The airline reserves at least 20 seats for executive class. However, at least4 times as many passengers prefer to travel by economy class than by the executive class. Determine how many tickets of each type must be sold in order to maximize the profit what is the max profit ?

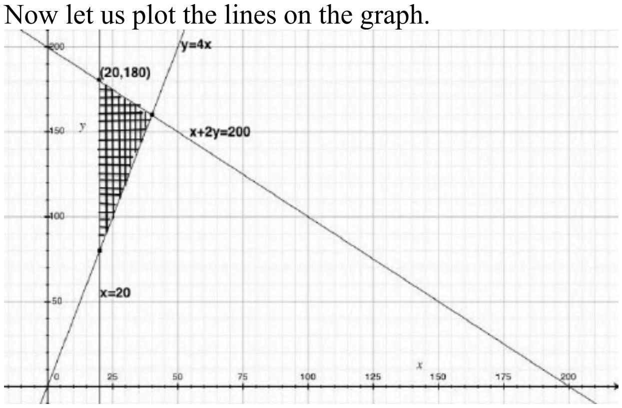
Sol.2) Step 1:
Let the executive class air tickets and economy class tickets sold be x and y
Now as the seating capacity of the aeroplane is 200,
so $x + y \leq 200$
As 20 tickets for executive class are to be resolved,
So we have $x \geq 20$
And as the number of tickets of economy class should be at least 4 times that of executive class $y \geq 4x$
Profit on sale of x tickets of executive class and y tickets of economy class $Z = 1000x + 600y$
Therefore LPP is (i.e) maximize $Z = 1000x + 600y$

subject to constraints

$x + y \leq 200, x \geq 20\ , y \geq 4x$ and $x, y \geq 0$

Step 2:

Now let us plot the lines on the graph.



$x = y = 200, x = 20$ and $y = 4x$

The region satisfying the inequalities $x + y \leq 200, x \geq 20$ and $y \geq 4x$ is ABC and it is shown in the figure as the shaded portion.

Step 3:

$Z = 100x + 600y$

The corner points of the feasible region A(20,180), B(40,160), C(20,80)

The values of the objective function at these points are as follows:

At the Points (x, y) the value of the objective function

subject to constraints

$z = 1000x + 600y$

At A(20,180) value of the objective function

$Z = 1000x + 600y \Rightarrow 1000 \times 20 + 600 \times 180 = 20000 + 108000 = 128000$

At B(40,160) value of the objective function

$Z = 1000x + 600y \Rightarrow 1000 \times 40 + 600 \times 160 = 40000 + 96000 = 136000$

At C(20,80) value of the objective function

$Z = 1000x + 600y \Rightarrow 1000 \times 20 + 600 \times 80 = 20000 + 48000 = 68000$

Step 4:

It is clear that at B(40,160) Z has the maximum value.

Hence $x = 40, y = 160$

This implies 40 tickets of executive class and 160 of economy class should be sold to get the maximum profit of $Rs. 136000$ ans.

Q.3) A toy company manufactures two types of dolls A and B. market tests and available resources indicated that the combined production level should not exceed $1200\ dolls$ per week and the demand for doll of type B is at most half of that for dolls of type A. Further the production level of dolls of type A can exceed three times the production of dolls of other type by at most $600\ units$. If the company makes profit of Rs.12 and Rs.16 per doll respectively on doll A and B. how many of each should be produced to max the profit ?

Sol.3) Let x & y be the number of dolls of type A & N respectively that are produced per week

The given problem can be formulated as follows:

Maximize $= 12x + 16y$ ……… (i)

Subject to the constraints

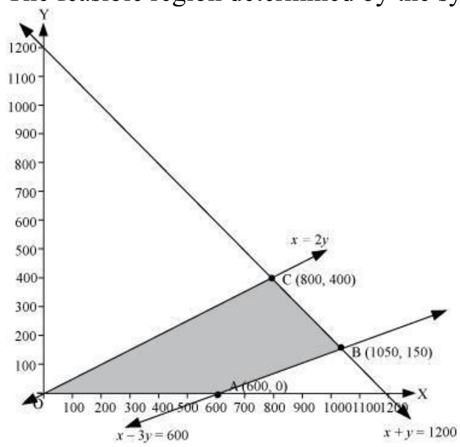
$x + y \leq 1200$ ……………. (ii)

$y \leq \frac{x}{2} \Rightarrow x \geq 2y$ ………….. (iii)

$x - 3y \leq 600$ ………….. (iv)

$x, y \geq 0$ ………….. (v)

The feasible region determined by the system of constraints as as follows:



The corner points are A(600, 0), B(1050, 150), C(800, 400)

The values of z at these corner points are as follows:

Corner point	$z = 12x + 6y$
A(600 , 0)	7200
B(1050, 150)	15000
C(800, 400)	16000 (maximum)

The maximum value of z is 16000 at (800, 400)

Thus, 800 & 400 dolls type A & type B should be produced respectively to get the maximum profit of Rs. 16000

Q.4) A factory owner purchases two types of machines A and B for his factory. The requirements and limitation for the machines are as follows :

	Area occupied by the machine	Labour force	Daily output
Machine A	$1000\ sq.m$	$12\ men$	60
Machine B	$1200\ sq.m$	$8\ men$	40

He has an area of $7600\ sq.m$ available and 72 skilled men who can operate the machines. How many machines of each type should he buy to maximize the daily output ?

Sol.4) Let x & y be the number of machines A & B respectively

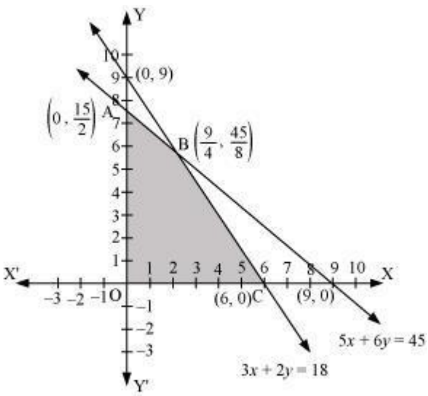
Now, according to given information, the mathematical formulation of the given problem is:

Maximize $Z = 60x + 40y$

Subject to the constraints

$1000x + 1200y \leq 9000$
$\Rightarrow 5x + 6y \leq 45$ ………… (i)
$12x + 8y \leq 72$
$\Rightarrow 3x + 2y \leq 18$ ………… (ii)
$x \geq 0, y \geq 0$ ………… (iii)
The inequalities (i) to (iii) can be graphed as:

It is seen that the shaded portion $OABC$ is the feasible region & values of Z at the corner points are given by the following table:
We can find the value of Z at vertices 0, A, B & C as follows:



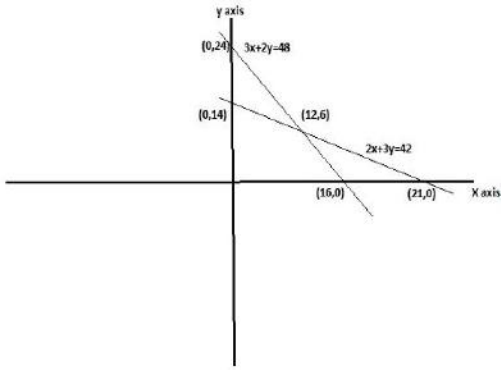
Corner point	$Z = 60x + 40y$
O (0, 0)	0
A $\left(0, \frac{15}{2}\right)$	300
B $\left(\frac{9}{4}, \frac{45}{8}\right)$	360 → maximum
C (6, 0)	360 → maximum

The maximum value of Z is $360\ units$, which is attained at B $\left(\frac{9}{4}, \frac{45}{8}\right)$ & C (6, 0).
It is clear that the number of machines cannot be in fraction.
Thus, to maximize the daily output, 6 machines of the type need to be bought.

Q.5) A library has to accommodate two different types of books on a shelf. The books are 6 cm and 4 cm thick and weight 1 kg and 1½ kg each respectively. The shelf is 96 cm long and at most can support a weight of 21 kg. How should be the shelf be find with books if two types in order to include the greater number of books ?

Sol.5) Let the number of books be x & y
Since the total length of the self is $96\ cm$. So,
$6x + 4y = 96$
$\Rightarrow 3x + 2y = 48$ ………… (i)
Since the total weight of the book shelf can accommodate is $21\ kg$. So,
$x + \frac{3}{2}y = 21$
$\Rightarrow 2x + 3y = 42$ ………… (ii)
We can simply get the answer by solving the above two linear equations, we get
$x = 12$ & $y = 6$

Using linear programming we need to plot the linear equations mentioned in the equation (i) and (ii). the point of intersection of the two linear equation will give the solution. The graph will look like:



12 books of 1st type and 6 books of 2nd type ans.

Q.6) Kellogg is a new cereal formed of a mixture of bran and rice that contains at least $88\ gm$ of protein and at least 36 milligram of iron knowing that bran contains 80 grams of protein and 40 milligram of iron per kilogram and that rice contains 100 grams of protein and 30 milligram of iron per kilogram. Find the minimum cost Rs.5 per kg and rice cost $Rs. 4\ per\ kg$.

Sol.6) Let the cereal contains $x\ kg$ of bran and $y\ kg$ of rice.

Clearly $x \geq 0$ and $y \geq 0$.

According to the hypothesis the linear programming problem is :

Minimize $Z = 5x + 4y$

Subject to the constraints :

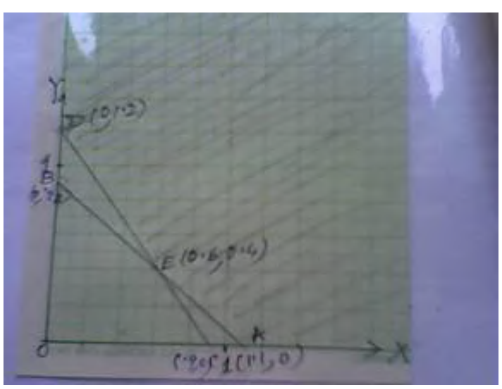
Or, $20\,x + 25\,y \geq 22$ ……….. (i)

And

Or, $20\,x + 15\,y \geq 18$ ……….. (ii)

And also $x \geq 0, y \geq 0$ ………… (iii)

Now the lines $20\,x + 25\,y = 22$ and $20x + 15\,y = 18$ are drawn.



These lines meet at E (0.6, 0.4). The feasible region is shaded and it is an unbounded region with vertices A (1.1, 0), E (0.6, 0.4) and D (0, 1.2).

Corner Point	$Z = 5\,x + 4\,y$	

$A\ (1.1, 0)$	$5 \times 1.1 + 4 \times 0 = 5.5$	
$E\ (0.6, 0.4)$	$5 \times 0.6 + 4 \times 0.4$ $= 4.6 \leftarrow$	Minimum
$D\ (0, 1.2)$	$5 \times 0 + 4 \times 1.2 = 4.8$	

∴ minimum cost of producing this cereal is $Rs. 4.60\ per\ kg.$ ans.

Q.7) A fruit grower can use two types of fertilizer in his garden brand P and brand Q. The amount (in kg) of nitrogen, phosphoric acid, potash and chlorine in a bag of watch brand are given in the table :-

	Kg per. Bag	
	Brand P	Brand Q
Nitrogen	3	3.5
Phosphoric acid	1	2
Potash	3	1.5
Chlorine	1.5	2

Tests indicate the garden needs at least 240kg of phosphoric acid, at least 270 kg of potash and at most 310 kg of chlorine. If the grower wants to minimize the amount of nitrogen added to the garden. How many bags of each brand should be used? What is the minimum amount of nitrogen added ?

Sol.7) Step 1:
Let x bags of brand P and y bags of brand Q are needed to minimize nitrogen contents.
∴ the objective function is to minimize.
$Z = 3x + 3.5y$
We need at least $240kg$ of phosphoric acid
$\therefore x + 2y \geq 240$

We require at least of $270kg$ of potash
$\therefore 3x + 1.5y \geq 270$

We require at most of $310kg$ of chlorine
$\therefore 1.5x + 3x \geq 310$

Hence the objective function is $Z = 3x + 3.5y$

Subject to constraints
$x + 2y \geq 240, 3x + 1.5y \geq 270, 1.5x + 2y \leq 310, x, y \geq 0$

Step 2:
Now let us draw the graph for the above equations,

(i) the line $x + 2y = 240$ passes through the point A(240,0) and (0,120)
Put $x = 0, y = 0$ in $x + 2y \geq 240$,we get $0 \geq 240$ which is not true.
This implies the region $x + 2y \geq 240$ lies on and above AB

(ii) Consider the line $3x + 1.5y \geq 270$
Clearly this passes through C(90,0), D(0,180)

Put $x = 0, y = 0$,we get
$0 \geq 270$ which is not true.
This implies the region $3x + 1.5y \geq 270$ lies on CD and above it.

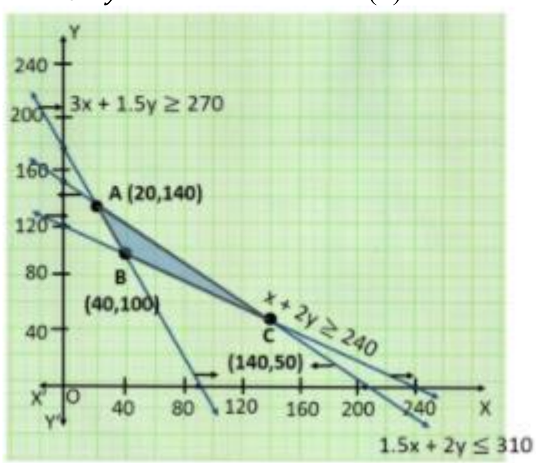
(iii)$1.5x + 2y \leq 310$
Put $x = 0, y = 0$
$\Rightarrow 0 \leq 310$ which is not true.
$\Rightarrow$ region $1.5x + 2y \leq 310$ lies on and below EF

(iv) $x \geq 0$ lies on and to the right of y-axis

(v) $y \geq 0$ lies on and above the x-axis.

Step 3:

The shaded area PP and RR represents the feasible region.
Now PP is the point of intersection of the lines,
$3x + 1.5y = 270$ …….. (i)
$1.5x + 2y = 310$ …….. (ii)



Let us solve the above equation to find the co-ordinate of P
Multiply equation (ii) by 2
$3x + 1.5y = 270$
$-3x + 4y = 620$
$-2.5y = -350$
$y = 140$

$\therefore\ x = 20$
Hence the coordinate of P are (20,140)

Step 4:
Q is the point of intersection of the line
$x + 2y = 240$ ....… (iii)
$3x + 1.5y = 270$ ………. (iv)
Multiply equation (iii) by 3
$3x + 6y = 720$
$-3x + 1.5y = 270$

$4.5y = 450$
$y = 100$
$\therefore x = 40$
Hence the coordinates of Q is (40,100)

Step 5:

R is the point of intersection of lines
$x + 2y = 240$
$-1.5x + 2y = 310$
$-0.5x = -70$
$x = 140$
$\therefore y = 50$
Hence the coordinates of RR is (140,50)
Hence the corner points of the feasible region is P(20,140), Q(40,100), R(140,50)

Step 6:

Now let us calculate the minimum value of Z as follows :
At the points (x,y) the value of the objective function subject to $Z = 3x + 3.5y$
At point P(20,140) the value of the objective function $Z = 3 \times 20 + 3.5 \times 140 = 550$
At point Q(40,100) the value of the objective function $Z = 3 \times 40 + 3.5 \times 100 = 470$
At point R(140,50) the value of the objective function $Z = 3 \times 140 + 3.5 \times 50 = 595$
It is clear that Z has a minimum value of 470 at Q(40,100)

Hence, 40 bags of brand P and 100 bags of brand Q
& minimum amount of nitrogen to be added is $470kg.$ ans.

Q.8) There are two types of fertilizers F_1 and F_2. F_1 consists of 10% nitrogen and 6% phosphoric acid and F_2 consists of 5% nitrogen and 10% phosphoric acid. After testing soil conditions, a farmer finds that she needs at least 14 kg of nitrogen 14 kg of phosphoric acid for crop. If F_1 costs Rs.6 / kg and F_2 costs Rs.5/kg. Determine how much of each type of fertilizer should be used so that the cost is minimum ?

Sol.8) Let the farmer buy $x\ kg$ of fertilizer F1 and $y\ kg$ of fertilizer F2 should be used and Z be the total cost
$\therefore x \geq 0\ \&\ y \geq 0$
Given

	Nitrogen (%)	Phosphoric acid (%)	Cost (Rs/kg)
$F_1(x)$	10	6	6
$F_2(y)$	5	10	5
Requirement (kg)	14	14	

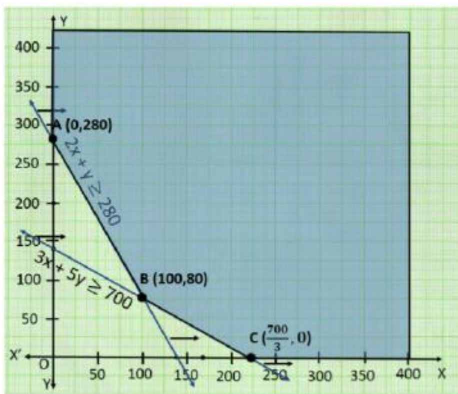
F_1 consists of 10% nitrogen & F_2 consists of 5% nitrogen. However, the farmer requires at least 14 kg of nitrogen.
$\therefore 10\%\ of\ x + 5\%\ of\ y \geq 14$
$\frac{x}{10} + \frac{y}{20} \geq 14$
$2x + y \geq 280$

F_1 consists of 6% phosphoric acid & F_2 consists of 10% phosphoric acid. However, the farmer requires at least 14 kg of phosphoric acid.
$\therefore 6\%\ of\ x + 10\%\ of\ y \geq 14$
$\frac{6x}{100} + \frac{10y}{100} \geq 14$

$3x + 56y \geq 700$

Total cost of fertilizers, $Z = 6x + 5y$
The mathematical formulation of the given problem is
Minimize $Z = 6x + 5y$ …….. (i)
Subject to the constraints
$2x + y \geq 280$ …….. (ii)
$3x + 5y \geq 700$ …….. (iii)
$x, y \geq 0$ …….. (iv)

The feasible region determined by the system of constraints is as follows:



It can be seen that the feasible region is unbounded. The corner points are
$A\left(\frac{700}{3}, 0\right), B(100, 80)$ & $C(0, 280)$

The values of Z at these points are as follows:

Corner point	$\boldsymbol{Z = 6x + 5y}$	
$A\left(\frac{700}{3}, 0\right)$	1400	
$B(100, 80)$	1000 →	Minimum
$C(0, 280)$	1400	

As the feasible region is unbounded, therefore, 1000 may or may not be the minimum value of Z
We draw graph of the inequality, $6x + 5y \leq 1000$, & check whether the resulting half plane has points in common with the feasible region or not.
It can be seen that the feasible region has no common point with
$6x + 5y < 1000$

$\therefore$ 100 kg of fertilizer F_1 & $80kg$ of fertilizers F_2 should be used to minimize the cost.
The minimum cost is $Rs. 1000$ ans.

Q.9) A cooperative society of farmers has 50 hectors of land to grow two crops X and Y. the profit from crops X and Y per hector are estimated as Rs. 10,500 and Rs. 9,000 respectively. To control weeds, a liquid herbicide has to be used for crops X and Y at rates of 20 liters and10 liters per hector. Further, no more than 800 liters of herbicide should be used in order to protect fish and wild life. How much land should be allocated to each crop so as to maximize the total profit ?

Sol.9) Let the land allocated for crop A be x & for crop B be y hectares

Maximum area of land available for two crops is 50 hectares

$x + y \leq 50$

Liquid herbicide to be used for crops A & B are at the rate of 20 litres & 10 litres per hectare respectively.

Maximum amount of herbicide to be used is 800 litres

$20x + 10y \leq 800$

$2x + y \leq 80$

The profits from crops A & B per hectare are Rs. 10,500 & Rs. 9,000 respectively

Thus, total profit $= Rs.(10500x + 9000y0 = Rs. 1500(7x + 6y)$

Therefore, the mathematical formulation of the given problem is

Maximize $Z = 1500(7x + 6y)$ subject of constraints

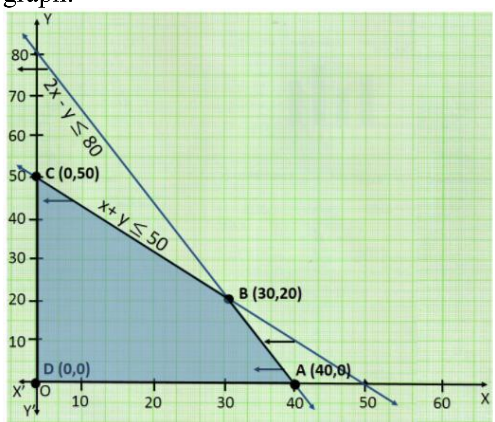
$x + y \leq 50$ .... (i)

$2x + y \leq 80$ .... (ii)

$x \geq 0$ .... (iii)

$y \geq 0$ .... (iv)

The feasible region determined by constraints (i), (ii), (iii) & (iv) is represented by the shaded region in the following graph:



The corner points of feasible region are O(0,0), A(40,0) , B(30,2) & C(0,50)

The values of Z are as follows:

Corner points	$Z = 1500(7x + 6y)$	
O(0,0)	0	
A(40,0)	420000	
B(30,2)	495000 →	Maximum
C(0,50)	420000	

The maximum profit is at point B(30,20)

Therefore, Max profit is Rs.4,95,000 by allocating 30 hector of land to crop A and 20 hectors of land to crop B. ans.

Q.10) A factory makes tennis rackets and cricket bats. A tennis racket takes 1.5 hrs of machine time and 3 hrs of craftsman time. While a cricket bat takes 3hr of machine time and 1 hrs of craftsman time. In a day 42 hrs of machine time and 24 hrs of craftsman time. What number of rackets and bats must be made if the factory is to work at full capacity ?

Sol.10) We are given 42h or machine time and 24h of craftsmen time.

Let x be the number of tennis rackets and y be the number of cricket bats that we can make.

Clearly, $x, y \geq 0$. Let us construct the following table from the given data.

	Tennis Rackets	**Cricket Bats**	**Requirements**

	(x)	(y)	
Machine Time (h)	1.5	3	42
Craftsmen Time (h)	3	1	24
Profit (Rs)	20	10	

Since we have only 42h of machine time and 24h of craftsmen time, we have to maximize $x + y$, given the following constraints:

$1.5x + 3y \leq 42$ .... (i)

$3x + y \leq 24$ .... (ii)

$x \leq 0$ .... (iii)

$y \leq 0$ .... (iv)

Plotting the constraints:

Plot the straight lines $1.5x + 3y = 42$ and $3x + y = 24$

First draw the graph of the line $1.5x + 3y = 42$.

If $x = 0, y = 14$ and if $y = 0, x = 28$.
So, this is a straight line between (0,14) and (28,0).
At (0,0), in the inequality, we have 0 + 0 = 0 which is ≤ 0.
So the area associated with this inequality is bounded towards the origin.
Similarly, draw the graph of the line $3x + y = 24$.
If $x = 0, y = 24$ and if $y = 0, x = 8$.
So, this is a straight line between (0,24) and (8,0).
At (0,0), in the inequality, we have 0 + 0 = 0 which is ≤ 0.
So the area associated with this inequality is bounded towards the origin.

Finding the feasible region :
We can see that the feasible region is bounded and in the first quadrant.
On solving the equations $1.5x + 3y = 42$ and $3x + y = 24$, we get,

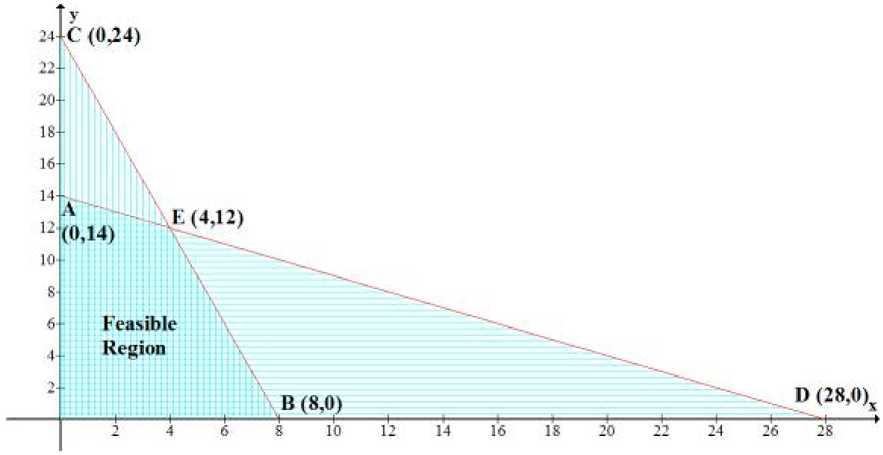
$1.5x + 3(24 - 3x) = 42 \Rightarrow 1.5x + 72 - 9x = 42$

$\Rightarrow -7.5x = -30 \Rightarrow x = 4$.

If $x = 4, y = 24 - 12 = 12$.

$\Rightarrow x = 4, y = 12$

Therefore the feasible region has the corner points (0,0), (0,14), (4,12), (8,0) as shown in the figure.



Solving the objective function using the corner point methodSolving the objective function using the corner point method:

The values of Z at the corner points are calculated as follows:

Corner Point	$Z = 20x + 10y$
O (0,0)	0
A (0,14)	160
E (4,12)	200 (Maximum value)
B (8,0)	140

A) The number of rackets and bats we can make at full capacity are 4 and 12.

B) The maximum profit we can make that meet the constraints is 200. Ans.