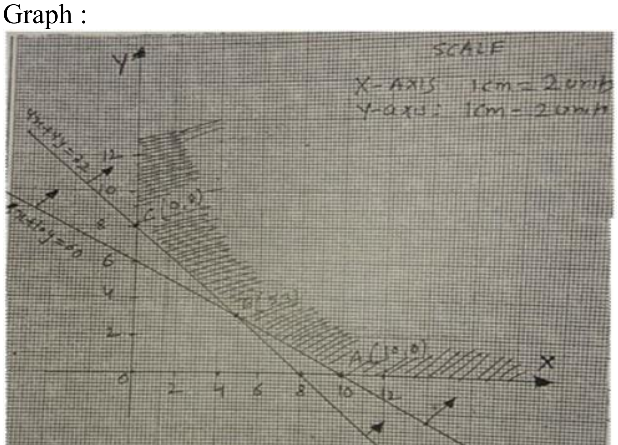
Linear Programming (LPP)

Class 12th

Q.1) Two tailors A and B earn Rs.150 and Rs.200 per day respectively. 'A' can stitch 6 shirts and 4 pants per day. While 'B' can stitch 10 shirts and 4 pants per day. Form LPP to minimize the labour cost to produce at least 60 shirts and 32 pants, and solve it.

Sol.1) Let x days taken by tailor A and y days taken by tailor B
let z be the total labour cost
LPP is given by ,
Minimize ….{labour cost}
$z = 150x + 200y$
subject to constraints ,
$6x + 10y \geq 60$ …..(shirt constraint)
$4x + 4y \geq 32$ …..(pant constraint)
and $x \geq 0 \; ; \; y \geq 0$
points : Ist Constraint : (0 , 6) and (10 , 0)
Solution : away from origin
IInd Constraint : (0 , 8) and (8 , 0)
Solution : away from origin
Graph :



Corner points	Value of the objective function $z = 150x + 200y$
$A(10\,,0)$	$z = 1500 + 0 = 1500$
$B(5\,,3)$	$z = 750 + 600 = 1350 \leftarrow$
$C(0\,,8)$	$z = 0 + 1600 = 1600$

∴ z is min at $x = 5$ and $y = 3$
∴ 5 days taken by tailor A and 3 days taken by tailor B to minimize the labour cost and minimize labour cost = $Rs.1350$ ans.

Q.2) A dealer wishes to purchase a number of fans and sewing machines. He has only Rs.5760 to invest and has space for at most 20 items. A fan costs him Rs. 360 and a sewing machine Rs.240. He can sell a fan at a profit of Rs.22 and a sewing machine at a profit of Rs. 18. How should he invest his money in order to maximize his profit.

Sol.2) Let dealer buys $'x'$ no of fans and $'y'$ no of machines.
Let Z be the total profit
LPP is given by ,
Maximize …...(profit)

$z = 22x + 18y$

subject to constraints

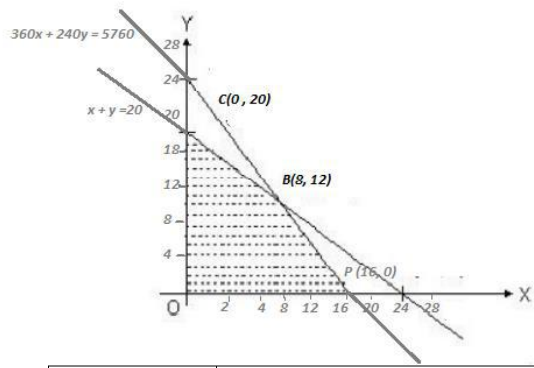
$360x + 240y < 5760$ …..(Investment constraint)

$x + y \leq 20$ …..(space available constraint)

and $x \geq 0$ and $y \geq 0$

points : I[st] Constraint : (0 , 24) and (16 , 0)
Solution : towards the origin
II[nd] Constraint : (0 , 20) and (20 , 0)
Solution : towards the origin

Graph :



Corner points	Value of the objective function $z = 22x + 18y$
$A(16, 0)$	$z = 22 \times 16 + 0 = 352$
$B(8, 12)$	$z = 22 \times 8 + 18 \times 12 = 392 \leftarrow$
$C(0, 20)$	$z = 0 + 18 \times 20 = 360$

$\therefore$ z is Maximize at $x = 8$ & $y = 12$
$\therefore$ dealer should purchase 8 fans and 12 sewing machines to max. the profit and max. profit = Rs.392 ans.

Q.3) A manufacturer of a line of patent medicines is preparing a production plan on medicines A and B. There are sufficient ingredients available to make 20,000 bottles of A and 40,000 bottles of B but there are only 45,000 bottles in to which either of the medicines can be put. Furthermore, it takes 3 hours to prepare enough material to fill 1000 bottles of A, it takes one hour to prepare enough material to fill 1000 bottle of B and there are 66 hours available for this operation. The profit is Rs. 8 per bottle for A and Rs. 7 per bottle for B formulate this problem as LPP and solve it.

Sol.3) Let manufacturer problem x bottle of medicine A and y bottle of B and let Z be the total profit
LPP is given by :
Maximize …..(profit)

$z = 8x + 7y$

subject to constraints ,

$x + y \leq 45{,}000$ …..(combined production constraint)

$x \leq 20{,}000$ …..(bottle A constraint)

$y \leq 40{,}000$ …..(bottle B constraint)

$\frac{3x}{1000} + \frac{1y}{1000} \leq 66$ ….(time available constraint)

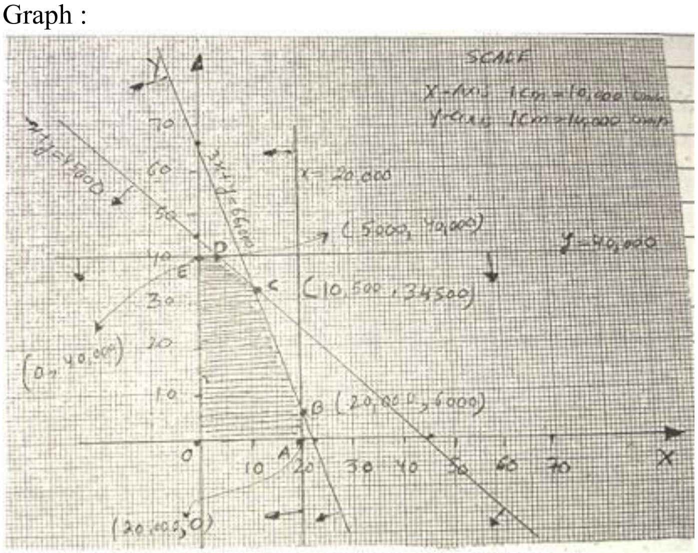
(or) $3x + y \leq 66{,}000$

and $x \geq 0$; $y \geq 0$

points : I[st] Constraint : points (0 , 45000) (45000 , 0)
Solution : towards the origin
II[nd] Constraint : points (20000 , 0) : ||[r] to y-axis
Solution : towards the origin
III[rd] Constraint : point (0- , 40000) : ||[r] to x-axis
Solution : towards the origin
IV[th] Constraint : points (0 , 66000) , (22000 , 0)
Solution : towards the origin

Corner points	Value of the objective function $z = 8x + 7y$
$A\,(20000\,,0)$	$z = 1{,}60{,}000$
$B\,(20000\,,6000)$	$z = 1{,}60{,}000 + 42{,}000$ $= 2{,}02000$
$C\,(10500\,,34500)$	$z = 84{,}000 + 2{,}41{,}500$ $= 325500$
$D\,(5000\,,40000)$	$z = 40{,}000 + 2{,}80{,}000$ $= 3{,}20{,}000$
$E\,(0\,,40000)$	$z = 2{,}80{,}000$

Graph :



z is maximizing at $x = 10{,}500$ & $y = 34{,}500$

∴ manufacture should fill 10,500 bottles of medicine A and 34,500 bottles of medicine B and the maximum profit is $Rs. 32{,}5500$ ans.

Q.4) Every gram of wheat provides $0.1\ gm$ of proteins and $0.25\ gm$ of carbohydrates. The corresponding values of rice are $0.05\ gm$ and $0.5\ gm$ respectively. Wheat cost $Rs.4\ per\ kg$ and rice Rs. 6 per kg. The minimum daily requirement of proteins and carbohydrates are $50gm$ and $200gm$ respectively. In what quantities should wheat and rice be mixed to provide minimum daily requirements of proteins and carbohydrates at minimum cost.

Sol.4) Let $x\ gm$ of wheat and $y\ gm$ of rice are mixed in daily diet

LPP

Minimize …..(cost)

$z = \frac{4x}{1000} + \frac{6y}{1000}$ …..{∵ x & y are in gram and cost in per kg}

subject to constraints ,

$0.1\,x + 0.05y \geq 50$

or $10x + 5y \geq 5000$ …...(protein constraint)

$0.25x + 0.5y \geq 200$

or $25x + 50y \geq 20000$ …...(carbohydrate constraint)

and $x \geq 0$, $y \geq 0$

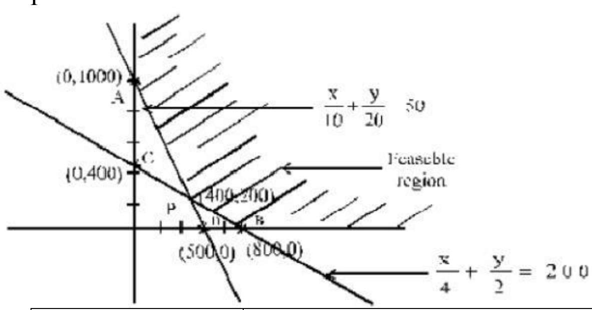
points : I[st] Constraint : (0 , 1000) , (500 , 0)

Solution : away from origin

II[nd] Constraint : (0 , 400) , (800 , 0)

Solution : away from origin

Graph :



Corner points	Value of the objective function $z = \frac{4x}{1000} + \frac{6y}{1000}$
$A\ (800\,,0)$	$Z = 3.2 + 0 = 3.2$
$B\ (400\,,200)$	$Z = 1.6 + 1.2 = 2.8$
$C\ (0\,,1000)$	$Z = 0 + 6 = 6$

∴ z is Minimum at $x = 400$ & $y = 200$

∴ $400\ gm$ of wheat and $200gm$ of rice should be mixed and minimum cost is $Rs\ 2.8$ ans.

(Transportation Problem)

Q.5) An oil company has two depots A and B with capacities of 7000 liters and 4000 liters respectively. The company is to supply oil to three petrol pumps D, E, and F whose requirements are 4500, 3000 and 3500 liters respectively. The distance in (km) between the depots and patrol pumps is given in table :-

Distance (in km)		
To \ From	A	B

D	7	3
E	6	4
F	3	2

Assuming that the transportation cost per km is Rs. 1.00 per ten liters. How should the delivery be scheduled to minimize the transportation cost.

Sol.5) Let $'x'$ liters and $'y'$ liter be transported from depot A to D & E petrol pump respectively

Let 'Z' be the total transportation cost

Rate /cost of transportation is $Rs\ \frac{1}{(km)(10\ liters)}$

Total transportation cost is ,

$$z = \frac{7x}{10} + \frac{6y}{10} + \frac{(7000-x-y)3}{10} + \frac{(4500-x)3}{10} + \frac{(3000-y)4}{10} + \frac{(x+y-3500)2}{10}$$

$$z = \frac{1}{10}\ [7x + 6y + 21000 - 3x - 3y + 13500 - 3x + 12000 - 4y + 2x + 2y - 7000]$$

$$z = \frac{1}{10}(3x + y + 39500)$$

since quantities are always non-negative

$\therefore\ x \geq 0\ ;\ y \geq 0$

$7000 - x - y \geq 0 \ \Rightarrow x + y \leq 7000$

$4500 - x \geq 0 \quad \Rightarrow x \leq 4500$

$3000 - y \geq 0 \quad \Rightarrow y \leq 3000$

$x + y - 3500 \geq 0 \Rightarrow x + y \geq 3500$

LPP

Minimize …..(cost)

$$z = \frac{1}{10}(3\text{x} + y + 39500)$$

subject to constraints

$x + y \leq 7000$

$x \leq 4500$

$x \leq 3000$

$x + y \leq 3500$

$and\ x \geq 0\ ;\ y \geq 0$

points : Ist Constraint : (0 , 7000) , (7000 , 0)

Solution : toward the origin

IInd Constraint : (4500 , 0) $\|^{r}$ to y-axis

Solution : towards the origin

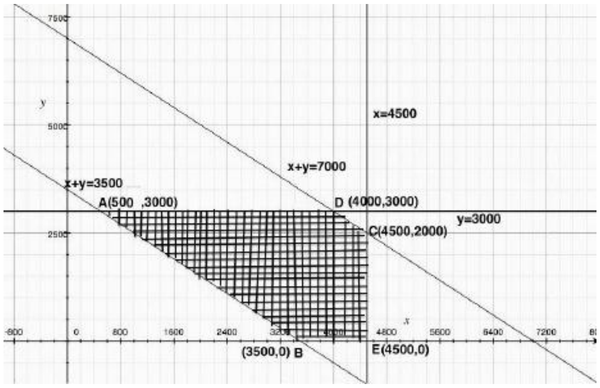
IIIrd Constraint : (0 , 3000) Parallel to x-axis

Solution : towards the origin

IVth Constraint : (0 , 3500) , (3500 , 0)

Solution : away from origin

Graph :



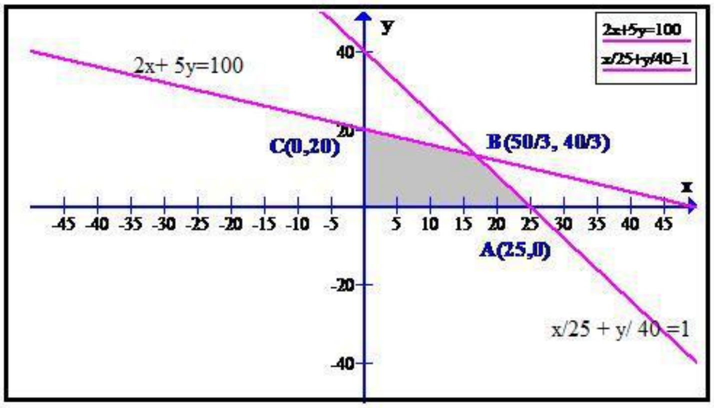
Corner Points	Value of objective function $z = \frac{1}{10}(3x + y + 39500)$
$A\ (3000\ ,0)$	$z\ =\ 4850$
$B\ (4500\ ,0)$	$z\ =\ 5300$
$C\ (4500\ ,2500)$	$z = \frac{1}{10}(13500 + 2500 + 39500)$ $= 5550$
$D\ (4000\ ,3000)$	$z = \frac{1}{10}(12000 + 3000 + 39500)$ $= 5450$
$E\ (500\ ,3000)$	$z = \frac{1}{10}(1500 + 3000 + 39500)$ $= 4400$

∴ z is Minimum at $x\ =\ 500$ & $y\ =\ 3000$
∴ From A : 500 liters , 3000 liters & 3500 liters to D, E, F Respectively
∴ From B : 4000 liters , 0 liter and 0 liter to D, E, F Respectively
and Min transportation cost is Rs. 4400 ans.

Q.6) If a young man drives his vehicle at , he has to spend $Rs. 2\ per\ km$ on petrol. If he drives it at a faster speed of , the petrol cost increases to . He has $Rs. 100$ to spend on petrol and travel with one hour. Express this on LPP and solve the same.

Sol.6) Let $x\ km$ distance traveled with speed and $y\ km$ distance traveled with speed.
Let z → total distance traveled.
LPP :
Maximize , (Distance)
$Z\ =\ x\ +\ y$
subject to constraints
$2x\ +\ 5y\ \leq\ 100$ …..(Investment on petrol constraint)
$\left(time = \frac{Distance}{speed}\right) \frac{x}{25} + \frac{y}{40} \leq 1$…..(time available constraint)
or $8x\ +\ 5y\ \leq\ 200$
and $x\ \geq\ 0\ ;\ y\ \geq\ 0$

points : I^{st} Constraint : (0 , 20) , (50 , 0)
Solution : toward the origin
II^{nd} Constraint : (0 , 40) (25 , 0)
Solution : towards the origin
Graph :



Corner point	Value of the objective function $z = x + y$
$A\ (25, 0)$	$Z = 25 + 0 = 25$
$B\left(\frac{50}{3}, \frac{40}{3}\right)$	$z = \frac{50}{3} + \frac{40}{3} = 30$
$C\ (0, 20)$	$Z = 0 + 20 = 20$

∴ Z is Max at $x = \frac{50}{3},\ y = \frac{40}{3}$
∴ Distance traveled at speed
∴ Distance traveled at speed
and Maximum Distance $= 30km$ ans.

Q.7) A company makes two kinds of leather belts A and B. But A is high quality belt and B is of lower quality. The respective profit are Rs. 4 and Rs 3 per belt. Each belt of type A requires twice as much time as a belt of type B and if all belts of types B the company could make 1000 belts per day. The supply of leather is sufficient for only 800 belts per day (both A and B combined). Belt A requires a fancy buckle and only 400 buckles per day are available. There are only 700 buckles available for belt B. What should be the daily production of each type of belt to maximum the profit.

Sol.7) Let company makes x belt of kind A and y belt of kind B per day
Let Z → total profit
Main Point :-
1000 belts of kind B can be made per day and kind A requires double time, So 500 belts of kind A can be made per day
∴ total time taken to produce x belts of A and y belt of B is $\left(\frac{x}{500} + \frac{y}{1000}\right)$
since company is making x belt of A and y belt of B in one day.
$$\therefore \frac{x}{500} + \frac{y}{1000} \le 1 \Rightarrow 2x + y \le 1000$$
LPP :
Maximize .....(profit)
$$z = 4x + 3y$$

subject to constraints,

$2x + y \leq 1000$

$x + y \leq 800$ (supply of leather constraints)

$x \leq 400$ (fancy buckle for A)

$y \leq 700$ (fancy buckle for B)

and $x \geq 0 , y \geq 0$

points : I[st] Constraint : (0 , 1000) , (500 , 0)

Solution : toward the origin

II[nd] Constraint : (0 , 800) (800 , 0)

Solution : towards the origin

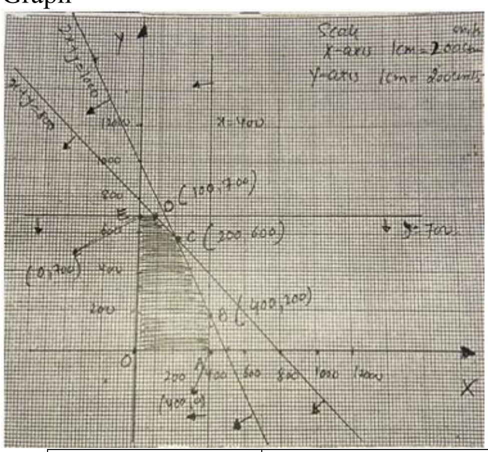
III[rd] Constraint : (400 , 0) ||[r] to y-axis

Solution : towards the origin

IV[th] Constraint : (0 , 700) ||[r] to x-axis

Solution : away from origin

Graph



Corner Points	Value of objective function $z = 4x - 3y$
$A\,(400\,,0)$	$Z = 1600 + 0 = 1600$
$B\,(400\,,200)$	$Z = 1600 + 600 = 2200$
$C\,(200\,,600)$	$Z = 800 + 1800 = 2600$ $\leftarrow$
$D\,(100\,,700)$	$Z = 400 + 1200 = 2500$
$E\,(0\,,700)$	$Z = 0 + 2100 = 2100$

Z is Max at $x = 200$ & $y = 600$

$\therefore$ company must produce 200 belts of kind A and 600 belts of kind B per day

and Max profit is Rs.2600 ans.

Q.8) A toy manufacturer produces two types of dolls ; a basic version doll A and deluxe version doll B. each doll of type B takes twice as long to produce as one doll of type A. The company have time to make a maximum of 2000 dolls of type A per day, the supply of plastic is sufficient to produce 1500 dolls per day. The deluxe version B requires a fancy dress of which there are only 600 per day available. If the company makes profit of Rs 3 and Rs 5 per doll on A & B respectively. How many of each should be produced to maximize the profit ?

Sol. 8) Let x dolls of type A and y dolls of type B are manufactured per day

Let z → total profit

LPP : Maximize ….(profit)

$$z = 3x + 5y$$

If t hours are requiring to produce one doll of type A then the time required to produce one doll of type B will be 2t hours

∴ total time consume, per day, to produce x & y dolls of type A & type B respectively is $xt + y2t \leq 2000t\ hrs.$ But the time available per day is $2000t\ hrs.$

$xt + y2t \leq 2000t$

Or $x + 2y \leq 2000$

Since plastic is available to produce 1500 dolls only

$\therefore x + y \leq 1500$

Also fancy dress is available for 600 dolls per day only

$\therefore y \leq 600$

Hence, the linear programming problem is as follows

Maximum

$Z = 3x + 5y$

Subject to constraints:

$x + 2y \leq 2000$

$x + y \leq 1500$

$y \leq 600$

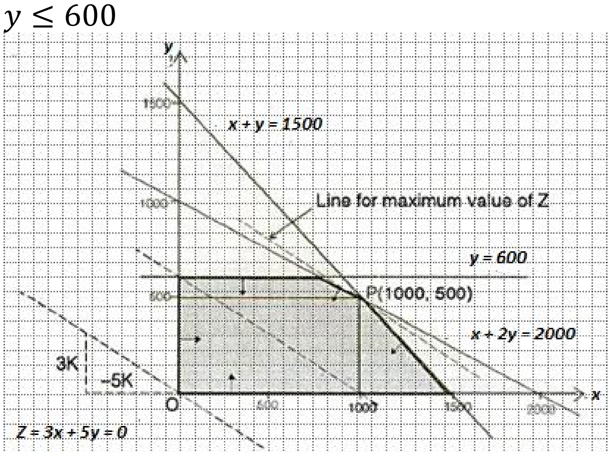
Step 1:

First we consider the constraints as equalities

$x + 2y \leq 2000$

$x + y \leq 1500$

$y \leq 600$



Step 2:

Here we draw lines in two dimensional plane

Step 3:

The shaded region in fig. is the permissible region for the values of the variables x & y

Step 4:

From the fig. it can be seen that Z is maximum at the point $P(1000, 500)$ which is the point of intersection

of the lines
$x + y = 2000$
$x + y = 1500$
∴ for maximum $Z, x = 1000$ & $y = 500$
& Maximum $Z = Rs.[3 \times 1000 + 5 \times 500] = Rs. 5500$

Hence, 1000 dolls of A and 500 dolls of type B and Max profit = Rs 5500 ans.

Q.9) Two godown A and B have grain storage capacity of 100 quintals and 50 quintals respectively. They supply to 3 ration shops D, E and F , whose requirements are 60, 50 and 40 quintals respectively. The cost of transportation per quintal from the god0wn to the shop are given in table

Transportation cost per quintal (in Rs)		
To \ From	A	B
D	6.00	4.00
E	3.00	2.00
F	2.50	3.00

how should the supplies be transported in order that the transportation cost is minimum ?

Sol.9) Step 1:

Let x quintals of grains be transported from godown A to ration shop D and y quintals of grains be transported from godown A to ration shop E.
Therefore the grains transported from godown A to shop F will be $100 - (x + y)$ quintals.
Because capacity of godown A is 100 quintals.
∴ we have $x \geq 0, y \geq 0$ and $100 - (x + y) \geq 0$ $(i.e)$ $x + y \leq 100$
As requirements of grains at shop D is 60 quintals, $(60 - x)$ quintals of grain will be transported from godown B to D

Step 2:

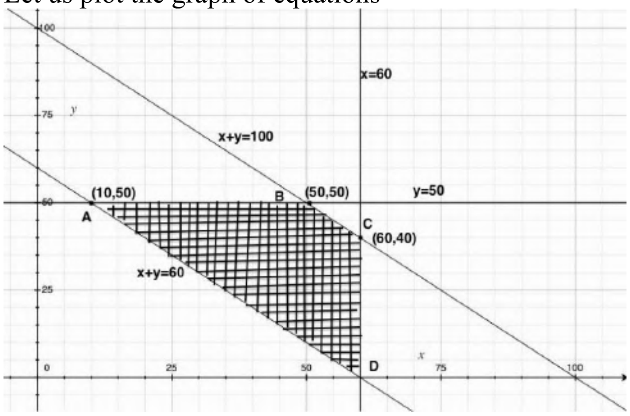
Similarly $(50 - y)$ quintals and $40 - [100 - (x + y)]$should be transported from godown B to shops E and F respectively.
Hence $(60 - x) \geq 0$
$\Rightarrow x \leq 60$
Similarly $(50 - y) \geq 0$
$\Rightarrow y \leq 50$
$40 - [100 - (x + y)] \geq 0$
$-60 + x + y \geq 0$
$x + y \geq 60$

Step 3:

The cost of transportation is
$Z = 6x + 3y + 250(100 - x - y) + 4(60 - x) + 2(50 - y) + 3(40 - 100 - x + y]$
Simplifying this we get,
$\Rightarrow 6x + 3y + 250 - 2.5x - 2.5y + 240 - 4x + 100 - 2y + 120 - 300 + 3x + 3y$
(i.e) $Z = 2.5x + 1.5y + 410$

Subject to constraints :
$x \leq 60, y \leq 50, x + y \geq 60, x + y \leq 100, y \geq 0$

Step 4:
Let us plot the graph of equations



$x = 60, y = 50, x + y = 60x = 60, and\ x + y = 100$

The feasible region ABCD shows the shaded area in the figure satisfies the inequalities
$x \leq 60, y \leq 50, x + y \geq 60, x + y \leq 100\ and\ x, y \geq 0$

Step 5:
Let us calculate the values of Z at the corner points $A(10,50), B(50,50), C(60,40)$ and $D(60,0)$
At the points (x, y) the value of the objective function
$Z = 2.5x + 1.5y + 410$
At A(10,50) the value of the objective function
$Z = 2.5 \times 10 + 1.5 \times 50 + 410 = 25 + 75 + 410 = 510$
At B(50,50) the value of the objective function
$Z = 2.5 \times 50 + 1.5 \times 50 + 410 = 125 + 75 + 410 = 610$
At C(60,40) the value of the objective function
$Z = 2.5 \times 60 + 1.5 \times 40 + 410 = 150 + 60 + 410 = 620$
At D(60,0) the value of the objective function
$Z = 2.5 \times 60 + 1.5 \times 0 + 410 = 150 + 0 + 410 = 560$

From A : 10 quintals , 50 quintals and 40 quintals to D, E, F respectively.
From B : 50 and 0 quintals to D, E, F respectively

This implies the minimum cost 510 at A(10,50) ans.

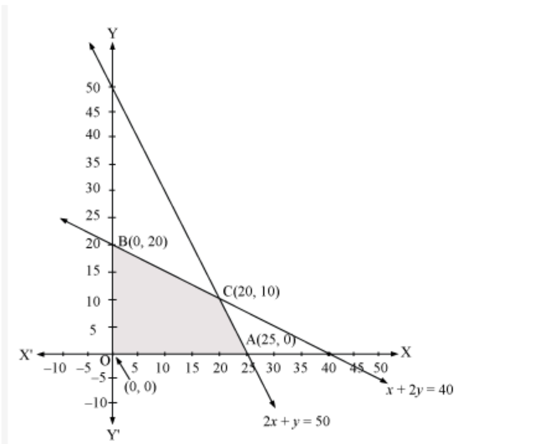
Q.10) One kind of cake requires $300\ gm$ of flour and $15\ gm$ of fat, another kind of cake requires $150\ gm$ of flour and $30\ gm$ of fat. Find the maximum number of cakes which can be made from $7.5\ kg$ of flour and $600\ gm$ of fat, assuming there is no shortage. Make it as on LPP and solve it.

Sol.10)

Types	Flour (in gm)	Fat (in gm)
I	300	15
II	150	30
Total	7.5 kg = 7500 gm	600 gm

Let the number of cakes of type I & type II be x & y respectively.
We need to maximize $z = x + y$
Subject to constraints
$300x + 150y \leq 7500 \Rightarrow 2x + y \leq 50$ ………… (i)
$15x + 30y \leq 600 \Rightarrow x + 2y \leq 40$ ………….. (ii)
Such that $x, y \geq 0$

The corner point of feasible region are O (0 , 0), A(25, 0) , C(20 , 10) & B(0 , 20)



The value of Z at the corner point can be calculated as

Corner point	Value of Z
0 (0 , 0)	0 + 0 = 0
A (25, 0)	25 + 0 = 0
B (0 , 20)	0 + 20 = 20
C (20, 10)	20 + 10 = 30 (maximum)

The maximum value of Z is 30
The maximum number of cakes which can be made is 30
20 cakes of type I and 10 cakes of type II. ans.