Integration (Indefinite Integrals)

Q.1)	(i) $I = \int \sin^2 x\, dx$ (ii) $I = \int \tan^2(2x) dx$ (iii) $I = \int \cot^2(3x) dx$ (iv) $I = \int \cos^2(4x) dx$
Sol.1)	(i) $I = \int \sin^2 x\, dx$ = $\frac{1}{2}\int 1 - \cos(2x) dx$ = $\frac{1}{2}\left[x - \frac{\sin(2x))}{2}\right] + c$ ans. (ii) $I = \int \tan^2(2x) dx$ = $\int sec^2(2x) - 1 dx$ = $\frac{\tan(2x)}{2} - x + c$ ans. (iii) $I = \int \cot^2(3x) dx$ = $\int cosec^2(3x) - 1 dx$ = $\frac{-1}{3}\cot(3x) - x + c$ ans. (iv) $I = \int \cos^2(4x) dx$ = $\frac{1}{2}\int 1 + \cos(8x) dx$ = $\frac{1}{2}\left[x + \frac{\sin(8x)}{8}\right] + c$ ans.
Q.2)	(i) $I = \int \sin^3 x dx$ (ii) $I = \int \cos^3(2x) dx$
Sol.2)	(i) $I = \int \sin^3 x dx$ = $\frac{1}{4}\int 3\sin x - \sin(3x) dx$ = $\frac{1}{4}\left[-3\cos x + \frac{\cos(3x)}{3}\right] + c$ ans. (ii) $I = \int \cos^3(2x) dx$ = $\frac{1}{4}\int 3\cos(2x) + \cos(6x) dx$ = $\frac{1}{4}\left[\frac{3}{2}\sin(2x) + \frac{1}{6}\sin(6x)\right] + c$ ans.
Q.3)	(i) $I = \int \tan^3 x dx$ (ii) $I = \int \cot^3(3x) dx$
Sol.3)	(i) $I = \int \tan^3 x dx$ = $\int \tan x . \tan^2 x dx$ = $\int \tan x . (sec^2 x - 1) dx$ = $\int \tan x . sec^2 x - \tan x dx$ = $\int \tan x\ . sec^2 x\, dx - \int \tan x\ dx$ put $\tan x = t$ in (I)...

	$\sec^2 x\ dx = dt$ $\therefore I = \int t\ dt - \log\mid secx \mid$ $= \frac{t^2}{2} - \log\mid secx \mid + c$ Replacing t by tan x $I = \frac{1}{2}\tan^2 x - \log\mid \sec x \mid + c$ ans. (ii)$I = \int \cot^3(3\mathrm{x})dx$ $= \int \cot(3\mathrm{x}).\cot^2(3\mathrm{x})dx$ $= \int \cot(3\mathrm{x})(cosec^2(3\mathrm{x}) - 1)dx$ $= \int \cot(3\mathrm{x}).cosec^2(3\mathrm{x}) - \cot(3\mathrm{x})dx$ $= \int \cot(3\mathrm{x}).cosec^2(3\mathrm{x})dx - \int \cot(3\mathrm{x})dx$ put cot(3x) = t in (I) $\therefore$ $-\mathrm{cosec}^2(3x)\ .\ 3\ dx = dt$ $cosec^2(3\mathrm{x})dx = \frac{-dt}{3}$ $\therefore I = -\frac{1}{3}\int t\ dt - \frac{1}{3}\log\lvert\sin(3\mathrm{x})\rvert$ $= \frac{-1}{6}t^2 - \frac{1}{3}\log\mid\sin(3\mathrm{x})\mid + c$ Replacing t $I = -\frac{1}{6}\cot^2(3\mathrm{x}) - \frac{1}{3}\log\mid\sin(3\mathrm{x})\mid + c$ ans.
Q.4)	(i) $I = \int \sin^4 x dx$ (ii)$I = \int \cos^4(2\mathrm{x})dx$
Sol.4)	(i) $I = \int \sin^4 x dx$ $= \int (\sin^2 x)^2 dx$ $= \int \left(\frac{1-\cos(2\mathrm{x})}{2}\right)^2 dx$ $= \frac{1}{4}\int 1 + \cos^2(2\mathrm{x}) - 2\cos(2\mathrm{x})dx$ $= \frac{1}{4}\int 1 + \frac{1+\cos(4\mathrm{x})}{2} - 2\cos(2\mathrm{x})dx$ $= \frac{1}{8}\int 2 + 1 + \cos(4\mathrm{x}) - 4\cos(2\mathrm{x})dx$ $= \frac{1}{8}\int 3 + \cos(4\mathrm{x}) - 4\cos(2\mathrm{x})dx$ $I = \frac{1}{8}\left[3\mathrm{x} + \frac{\sin(4\mathrm{x})}{4} - 2\sin(2\mathrm{x})\right] + c$ ans. (ii)$I = \int \cos^4(2\mathrm{x})dx$ $= \int (\cos^2(2\mathrm{x}))^2 dx$ $= \int \left(\frac{1+\cos(4\mathrm{x})}{2}\right)^2 dx$ $= \frac{1}{4}\int 1 + \cos^2(4\mathrm{x}) + 2\cos(4\mathrm{x})dx$ $= \frac{1}{4}\int 1 + \frac{1+\cos(8\mathrm{x})}{2} + 2\cos(4\mathrm{x})dx$ $= \frac{1}{8}\int 3 + \cos(8\mathrm{x}) + 4\cos(4\mathrm{x})dx$

	$= \ I = \frac{1}{8}\left[3x + \frac{\sin(8x)}{8} + \frac{4}{4}\sin(4x)\right] + c$ ans.
Q.5)	(i) $I = \int \tan^4 x dx$ (ii) $I = \int \cot^4(3x) dx$
Sol.5)	(i) $I = \int \tan^4 x dx$ $= \int \tan^2 x . \tan^2 x dx$ $= \int \tan^2 x . (sec^2 x - 1) dx$ $= \int \tan^2 x . sec^2 x dx - \int \tan^2 x dx$ $= \int \tan^2 x . sec^2 x dx - \int sec^2 x - 1 dx$ put tan $x = t$ $\therefore$ $\sec^2 x\, dx = dt$ $I = \int t^2 dt - (\tan x - x)$ $= \frac{t^3}{3} - \tan x + x + c$ $I = \frac{1}{3}\tan^3 x - \tan x + x + c$ ans. (ii) $I = \int \cot^4(3x) dx$ $= \int \cot^2(3x) . \cot^2(3x) dx$ $= \int \cot^2(3x) . (cosec^2(3x) - 1) dx$ $= \int \cot^2(3x) . cosec^2(3x) dx - \int \cot^2(3x) dx$ put $\cot(3x) = t$ $-3\ \mathrm{cosec}^2(3x)\ dx = dt$ $\therefore cosec^2(3x) dx = -\frac{dt}{3}$ $I = \frac{-1}{3}\int t^2 dt - \int cosec^2(3x) - 1 dx$ $= \frac{-1}{3} . \frac{t^3}{3} - (\frac{\cot(3x)}{3} - x) + c$ $I = \frac{-1}{9}\cot^2(3x) - \frac{\cot(3x)}{3} + x + c$ ans.
Q.6)	$I = \int sec^4 x\ dx$
Sol.6)	$I = \int sec^4 x\ dx$ $= \int sec^2 x . sec^2 x dx$ $= \int (1 + \tan^2 x) sec^2 x dx$ $= \int sec^2 x dx + \int \tan^2 x . sec^2 x dx$ put tan $x = t$ $\therefore\ sec\ x\ dx = dt$ $= \tan x + \int t^2 dt$ $= \tan x + \frac{t^3}{3} + c$ $= \ I = \tan x + \frac{\tan^3}{3} + c$ ans.
→	**<u>Sin *x* and Cos *x* in multiplication with same power</u> :-**
Q.7)	(i) $I = \int \sin^2 x . \cos^2 x dx$ (ii) $I = \int \sin^4 . \cos^4 x dx$

Sol.7)	(i) $I = \int \sin^2 x.\cos^2 x dx$ $= \int (\sin x . \cos x)^2 dx$ $= \int \left(\frac{\sin(2x)}{2}\right)^2 dx$ $= \frac{1}{4}\int \sin^2(2x) dx$ $= \frac{1}{4}\frac{\int 1-\cos(4x)}{2} dx$ $= \frac{1}{8}\int 1-\cos(4x) dx$ $= I = \frac{1}{8}\left[x - \frac{\sin(4x)}{4}\right] + c$ ans. (ii) $I = \int \sin^4 . \cos^4 x dx$ $= \int (\sin x . \cos x)^4 dx$ $= \int \left(\frac{\sin(2x)}{2}\right)^4 dx$ $= \frac{1}{16}\int \sin^4(2x) dx$ $= \frac{1}{16}\int (\sin^2(2x))^2 dx$ $= \frac{1}{16}\int \left(\frac{1-\cos(4x)}{2}\right)^2 dx$ $= \frac{1}{64}\int 1+\cos^2(4x) - 2\cos(4x) dx$ $= \frac{1}{64}\int 1+\frac{1+\cos(8x)}{2} - 2\cos(4x) dx$ $= \frac{1}{128}\int 3+\cos(8x) - 4\cos(4x) dx$ $= I = \frac{1}{128}\left[3x + \frac{\sin(8x)}{8} - \sin(4x)\right] + c$ ans.
→	**<u>Sin *x* and Cos *x* in multiplication with different power</u> :-**
Q.8)	(i) $I = \int \sin^3 x.\cos^4 x\, dx$
Sol.8)	(i) $I = \int \sin^3 x.\cos^4 x dx$ $= \int \sin^2 x.\cos^4 x.\sin x\, dx$ $= \int (1-\cos^2 x).\cos^4 x.\sin x\, dx$ put $\cos x = t$ $\therefore -\sin x\, dx = dt$ $\sin x\, dx = -dt$ $I = -\int (1-t^2)t^4 dt$ $= -\int t^4 - t^6 dt$ $= -\left[\frac{t^5}{5} - \frac{t^7}{7}\right] + c$ $I = -\left[\frac{\cos^5 x}{5} - \frac{\cos^7 x}{7}\right] + c$ ans.
Q.9)	(i) $I = \int \sin^3 x.\cos^5 x dx$ (ii) $I = \int \sin^5 x dx$ (iii) $I = \int \cos^7 x dx$
Sol.9)	(i) $I = \int \sin^3 x.\cos^5 x dx$

	$= \int \sin^2 x . \cos^5 x . \sin x dx$ put $cos\ x = t$ $sin\ x\ dx = - dt$ $\therefore I = -\int (1 - t^2) t^5 dt$ $= -\int t^5 - t^7 dt$ $= \frac{-t^6}{6} + \frac{t^8}{8} + c$ $= \frac{-\cos^6 x}{6} + \frac{\cos^8 x}{8} + c$ ans. (ii)$I = \int \sin^5 x dx$ $= \int \sin^4 x . \sin x dx$ $= \int (1 - \cos^2 x)^2 . \sin x dx$ put cos $x = t$ $sin\ x\ dx = - dt$ $\therefore I = -\int (1 - t^2)^2 dt$ $= -\int 1 + t^4 - 2t^2 dt$ $= -\left[t + \frac{t^5}{5} - 2\frac{t^3}{3}\right] + c$ $I = -\left[\cos x + \frac{\cos^5 x}{5} - \frac{2\cos^3 x}{3}\right] c$ ans. (iii)$I = \int \cos^7 x dx$ $= \int \cos^6 x . \cos x dx$ $= \int (\cos^2 x)^3 . \cos x dx$ $= \int (1 - \sin^2 x)^3 \cos x dx$ put sin $x = t$ cos $x\ dx = dt$ $\therefore I = \int (1 - t^2)^3 dt$ $= \int 1 - t^6 - 3t^2 + 3t^4 dt$ $= t - \frac{t^7}{7} - \frac{3t^3}{3} + \frac{3t^5}{5} + c$ $= I = \sin x - \frac{\sin^7 x}{7} - \sin^7 x + \frac{3}{5}\sin^5 x + c$ ans.
Q.10)	)$I = \int \cos^7 x dx$
Sol.10)	$I = \int \cos^7 x dx$ $= \int \cos^6 x . \cos x dx$ $= \int (\cos^2 x)^3 . \cos x dx$ $= \int (1 - \sin^2 x)^3 \cos x dx$ put sin $x = t$ cos $x\ dx = dt$ $\therefore I = \int (1 - t^2)^3 dt$ $= \int 1 - t^6 - 3t^2 + 3t^4 dt$ $= t - \frac{t^7}{7} - \frac{3t^3}{3} + \frac{3t^5}{5} + c$

	= $I = \sin x - \frac{\sin^7 x}{7} - \sin^7 x + \frac{3}{5}\sin^5 x + c$ ans.