Integration (Indefinite integral)

Q.1)	$I = \int \frac{x^2}{(a+bx)^2} dx$
Sol.1)	$I = \int \frac{x^2}{(a+bx)^2} dx$ put $a + bx = t$ $bdx = dt \Rightarrow dx = \frac{dt}{b}$ $\therefore I = \frac{1}{b}\int \frac{x^2}{t^2} dt$ $= \frac{1}{b}\int \frac{\left(t-\frac{a}{b}\right)^2}{t^2} dt$ $= \frac{1}{b}.\frac{1}{b^2}\int \frac{t^2+a^2-2at}{t^2} dt$ Separate $= \frac{1}{b^3}\int 1 + \frac{a^2}{t^2} - \frac{2a}{t} dt$ $= \frac{1}{b^3}\left[t - \frac{a^2}{t} - 2a\log \mid t \mid\right] + c$ $= \frac{1}{b^3}\left[(a+bx) - \frac{a^2}{a+bx} - 2a\log \mid a+bx \mid\right] + c$ ans.
→	**<u>Type</u>: <u>When degree of Numerator ≥ degree of Denominator then divide and write</u>** $\int \frac{N}{b} dx = \int \theta + \frac{R}{b} dx$
Q.2)	(i) $I = \int \frac{x^7}{x-1} dx$ (ii) $I = \int \frac{1}{x^{1/2}+x^{1/3}} dx$
Sol.2)	(i) $I = \int \frac{x^7}{x-1} dx$ clearly degree of N^r > degree of D^r (then divide) $\therefore I = \int \theta + \frac{R}{D} dt$ $= \int (x^6 - x^5 + x^4 - x^3 + x^2 - x + 1) - \frac{1}{x+1} dx$ $= \int \frac{x^7}{7} - \frac{x^6}{6} + \frac{x^5}{5} - \frac{x^4}{4} + \frac{x^3}{3} - \frac{x^2}{2} + x$ $= -\log \mid x+1 \mid + c$ ans. (ii) $I = \int \frac{1}{x^{1/2}+x^{1/3}} dx$ put $x = t^6$ ....{L.C.M of 2 & 3 = 6} $dx = 6t^5 dt$ $\therefore I = 6\int \frac{t^5 dt}{t^3+t^2}$ $= \int \frac{t^5}{t^2(t+1)} dt$ $= \int \frac{t^3}{t+1} dt$ Degree of N > degree of D (then divide) $= \int (t^2 - t + 1) - \frac{1}{t+1} dt$

	$I=\frac{t^3}{3}-\frac{t^2}{2}+t-\log\mid t+1\mid+c$ replacing $t$by $x^{1/6}$ $\therefore I=\frac{x^{1/2}}{3}-\frac{x^{1/3}}{2}+x^{1/6}-\log\mid x^{1/6}+1\mid+c$ ans.
Q.3)	(i) $I=\int\frac{e^{2x}-1}{e^{2x}+1}dx$ (ii)$I=\int\frac{\sqrt{\tan x}}{\sin x.\cos x}dx$ (iii)$I=\int 2^{2^{2^x}}.2^{2^x}.2^x dx$
Sol.3)	(i) $I=\int\frac{e^{2x}-1}{e^{2x}+1}dx$ take e^x common in N and D $=\int\frac{e^x(e^x-e^{-x})}{e^x(e^x+e^{-x})}dx$ put $e^x+e^{-x}=t$ $\therefore\ (e^x-e^{-x})dx=dt$ $\therefore\ I=\int\frac{dt}{t}$ $=\log\mid t\mid+c$ $I=\log\mid e^x-e^{-x}\mid+c$ ans. (ii)$I=\int\frac{\sqrt{\tan x}}{\sin x.\cos x}dx$ Divide N and D by $\cos^2 x$ $I=\int\frac{\frac{\sqrt{\tan x}}{\cos^2 x}}{\frac{\sin x.\cos x}{\cos^2 x}}dt$ $=\int\frac{\sqrt{\tan x}.sec^2 x}{\tan x}$ put $\tan x=t$ $\therefore\ sec^2 x dx=dt$ $\therefore\ I=\int\frac{\sqrt{t}}{t}dt$ $=\int\frac{1}{\sqrt{t}}dt$ $I=2\sqrt{t}+c$ $I=2\sqrt{\tan x}+c$ ans. (iii)$I=\int 2^{2^{2^x}}.2^{2^x}.2^x dx$ put $2^{2^{2^x}}=t$ $\therefore 2^{2^{2^x}}.\log 2.2^{2^x}.\log 2.2^x.\log 2 dx=dt$ $\Rightarrow 2^{2^{2^x}}.2^{2^x}.2^x.(\log)^3 dx=dt$ $\Rightarrow 2^{2^{2^x}}.2^{2^x}.2^x dx=\frac{dt}{(\log 2)^3}$ $\therefore I=\frac{1}{(\log 2)^3}\int dt$ $=\frac{1}{(\log 2)^3}t+c$ $I=\frac{1}{(\log 2)^3}.2^{2^{2^x}}.+c$ ans.
Q.4)	(i) $I=\int\frac{x^5}{\sqrt{1+x^3}}dx$ (ii)$I=\int 5^{x+\tan^{-1}x}.\left(\frac{x^2+1}{x^2+1}\right)dx$

	(iii) $I = \int \frac{e^{\sqrt{x}}.\cos(e^{\sqrt{x}})}{\sqrt{x}} dx$ (iv) $I = \int \frac{(x+1)e^x}{\sin^2(xe^x)} dx$
Sol.4)	(i) $I = \int \frac{x^5}{\sqrt{1+x^3}} dx$ $\int \frac{x^3.x^2}{\sqrt{1+x^3}} dx$ put $1 + x = t$ $3x^2 dx = dt$ $x^2 dx = \frac{dt}{3}$ $\therefore I = \frac{1}{3}\int \frac{x^3}{\sqrt{t}} dt$ = $\frac{1}{3}\int \frac{(t-1)}{\sqrt{t}} dt$ Separate = $\frac{1}{3}\int \sqrt{t} - \frac{1}{\sqrt{t}} dt$ = $\frac{1}{3}\left[\frac{2}{3}t^{3/2} - 2\sqrt{t}\right] + c$ = $\frac{1}{3}\left[\frac{2}{3}(1 + x^3)^{3/2} - 2\sqrt{(1 + x^3)}\right] + c$ ans. (ii) $I = \int 5^{x+\tan^{-1}x}.\left(\frac{x^2+1}{x^2+1}\right) dx$ Hint : put $x + \tan^{-1}x = t$ $1 + \frac{1}{1+x^2} dx = dt$ $\frac{5^{x+\tan-1x}}{\log 5} + c$ ans. (iii) $I = \int \frac{e^{\sqrt{x}}.\cos(e^{\sqrt{x}})}{\sqrt{x}} dx$ put $e^{\sqrt{x}} = t$ $\frac{e^{\sqrt{x}}}{2\sqrt{x}} dx = dt$ $e^{\sqrt{x}} \frac{dx}{\sqrt{x}} = 2dt$ $\therefore I = 2\int \cos t dt$ = $2\sin t + c$ = $2\sin(e^{\sqrt{x}}) + c$ ans. (iv) $I = \int \frac{(x+1)e^x}{\sin^2(xe^x)} dx$ put $xe^x = t$ $(xe^x + e^x)dx = dt$ $e^x(x + 1)dx = dt$ $\therefore I = \int \frac{dt}{\sin^2 t} dt$ = $\int cosec^2 t dt$ = $-\cot t + c$ = $-\cot(xe^x) + c$ ans.
Q.5)	(i) $I = \int \frac{1}{1+\tan x} dx$ (ii) $I = \int \frac{1}{1+\cot x} dx$

Sol.5)	(i) $I = \int \frac{1}{1+\tan x} dx$ $= \int \frac{1}{1+\frac{\sin x}{\cos x}} dx$ $= \int \frac{\cos x}{\cos x+\sin x} dx$ $= \frac{1}{2}\int \frac{2\cos x}{\cos x+\sin x} dx$ $= \frac{1}{2}\int \frac{\cos x+\cos x+\sin x-\sin x}{\cos x+\sin x} dx$ $= \frac{1}{2}\int \frac{(\cos x+\sin x)+(\cos x-\sin x)}{\cos x+\sin x} dx$ Separate $= \frac{1}{2}\int 1+\frac{\cos x-\sin x}{\cos x+\sin x} dx$ $= \frac{1}{2}\int 1.dx+\frac{1}{2}\int \frac{\cos x-\sin x}{\cos x+\sin x} dx$ put $\cos x+\sin x = t$ $(-\sin + \cos x)dx = dt$ $= \frac{1}{x}+\frac{1}{2}\int \frac{dt}{t}$ $I = \frac{1}{2}x+\frac{1}{2}\log \mid \cos x+\sin x \mid +c$ ans. (ii)$I = \int \frac{1}{1+\cot x} dx$ $= \int \frac{1}{1+\frac{\cos x}{\sin x}} dx$ $= \int \frac{\sin x}{\sin x+\cos x} dx$ $= \frac{1}{2}\int \frac{2\sin x}{\sin x+\cos x} dx$ $= \frac{1}{2}\int \frac{\sin x+\sin x+\cos x-\cos x}{\sin x+\cos x} dx$ $= \frac{1}{2}\int \frac{(\sin x+\cos x)}{\sin x+\cos x}+\frac{(\sin x-\cos x)}{\sin x+\cos x} dx$ $= \frac{1}{2}\int 1+\frac{\sin x-\cos x}{\sin x+\cos x} dx$ $= \frac{1}{2}x+\frac{1}{2}\int \frac{\sin x-\cos x}{\sin x+\cos x} dx$ put $\sin x+\cos x = t$ $(\cos x-\sin x)dx = dt$ $I = \frac{1}{2}x+\frac{1}{2}\int \frac{dt}{t}$ $= \frac{1}{2}x+\frac{1}{2}\log \mid \sin x+\cos x \mid +c$ ans.
Q.6)	(i) $I = \int \frac{e^{5\log x}-e^{4\log x}}{e^{3\log x}-e^{3\log x}} dx$ (ii)$I = \int (x^4+1)^{-1}.e^{3\log x} dx$ (iii)$I = \int e^{\log\sqrt{x}} dx$ (iv) $I = \int \frac{(a^x+b^x)}{a^x+b^x} dx$
Sol.6)	(i) $I = \int \frac{e^{5\log x}-e^{4\log x}}{e^{3\log x}-e^{3\log x}} dx$ $= \int \frac{e^{\log x^5}-e^{\log x^4}}{e^{\log x^3}-e^{\log x^2}} dx$ $= \int \frac{x^5-x^4}{x^3-x^2} dx$ ......{∵ $e^{\log x} = x$} $= \int \frac{x^4(x-1)}{x^2(x-1)} dx$

	$= \int x^2 dx$ $I = \frac{x^3}{3} + c$ ans. (ii)$I = \int (x^4+1)^{-1}.e^{3\log x} dx$ $= \int \frac{e^{\log x3}}{x^4+1} dx$ $= \int \frac{x^3}{x^4+1} dx$ put $x^4 + 1 = t$ $4x^3 dx = dt \Rightarrow x^3 dx = \frac{dt}{4}$ $\therefore I = \frac{1}{4}\int \frac{dt}{t}$ $= \frac{1}{4}\log \mid x^4+1 \mid + c$ ans. (iii)$I = \int e^{\log\sqrt{x}} dx$ $= \int \frac{\sqrt{x}}{x} dx$ $= \int \frac{1}{\sqrt{x}} dx$ $= 2\sqrt{x} + c$ ans. (iv) $I = \int \frac{(a^x+b^x)}{a^x b^x} dx$ $= \int \frac{a^{2x}+b^{2x}+2a^x.b^x}{a^x b^x} dx$ Separate $= \int \frac{a^{2x}}{a^x b^x} + \frac{b^{2x}}{a^x b^x} + \frac{2a^x b^x}{a^x b^x} dx$ $= \int \frac{a^x}{b^x} + \frac{b^x}{a^x} + 2dx$ $= \int \left(\frac{a}{b}\right)^x + \left(\frac{b}{a}\right)^x + 2dx$ $= I = \frac{\left(\frac{a}{b}\right)^x}{\log(a/b)} + \frac{\left(\frac{b}{a}\right)^x}{\log(b/a)} + 2x + c$ ans.
Q.7)	(i) $I = \int (2\tan x - 3\cot x)^2 dx$ (ii)$I = \int \frac{1}{\sin^2 x.\cos^2 x} dx$ (iii)$I = \int \tan^{-1}\sqrt{\frac{1-\sin x}{1+\sin x}} dx$ (iv) $I = \int \frac{\sin(2x)}{a^2\sin^2 + b^2\cos^2 x} dx$
Sol.7)	(i) $I = \int (2\tan x - 3\cot x)^2 dx$ $= \int 4\tan^2 x + 9\cot^2 x - 12\tan x.\cot x dx$ $= \int 4(sec^2 x - 1) + 9(cosec^2 x - 1) - 12dx$ $= 4(\tan x - x) + 9(-\cot x - x) - 12x + c$ $= 4\tan x - 9\cot x - 25x + c$ ans. (ii)$I = \int \frac{1}{\sin^2 x.\cos^2 x} dx$ $= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x.\cos^2 x} dx$ Separate: $= \int \frac{\sin^2 x}{\sin^2 x\cos^2 x} + \frac{\cos^2 x}{\sin^2 x\cos^2 x} dx$

	= $\int sec^2x + cosec^2x dx$ = $I = \tan x - \cot x + c$ ans. (iii) $I = \int \tan^{-1}\sqrt{\frac{1-\sin x}{1+\sin x}}dx$ = $\int \tan^{-1}\sqrt{\frac{1-\cos\left(\frac{\pi}{2}-x\right)}{1+\cos\left(\frac{\pi}{2}+x\right)}}dx$ = $\int \tan^{-1}\sqrt{\frac{2\sin^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}{2\cos^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}}dx$ = $\int \tan^{-1}\sqrt{\tan^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}dx$ = $\int \tan^{-1}\left(\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right)dx$ = $\int \frac{\pi}{4}-\frac{x}{2}dx$ $I = \frac{\pi x}{4}-\frac{x^2}{4}+c$ ans. (iv) $I = \int \frac{\sin(2x)}{a^2\sin^2x+b^2\cos^2x}dx$ put $a^2\sin^2x + b^2\cos^2x = t$ $a^2 2\sin x\cos x - b^2 2\cos x\sin x dx = dt$ $a^2\sin(2x) - b^2\sin(2x)dx = dt$ $\sin(2x)(a^2-b^2)dx = dt$ $\sin(2x)dx = \frac{dt}{a^2-b^2}$ $\therefore I = \frac{1}{a^2-b^2}\int \frac{dt}{t}$ = $\frac{1}{a^2-b^2}\log \mid t \mid + c$ = $I = \frac{1}{a^2-b^2}\log \mid a^2\sin^2x + b^2\cos^2x \mid + c$ ans.
Q.8)	(i) $I = \int \frac{\log\left(\tan\frac{x}{2}\right)}{\sin x}dx$ (ii) If $f'(x) = x + b, f(1) = 5$ and $f(2) = 13$. Find $f(x)$ (iii) If $f'(x) = 3x^2 - \frac{2}{x^3}$ and $f(1) = 0$. Find $f(x)$. (iv) $I = \int \frac{e^{x-1}+e^{-1}}{e^x+x^e}dx$
Sol.8)	i) $I = \int \frac{\log\left(\tan\frac{x}{2}\right)}{\sin x}dx$ put $\log\left(\tan\frac{x}{2}\right) = t$ $\frac{1}{\tan\frac{x}{2}}.sec^2\left(\frac{x}{2}\right).\frac{1}{2}dx = dt$ $\Rightarrow \frac{\frac{1}{\cos^2\left(\frac{x}{2}\right)}}{\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}}.\frac{1}{2}dx = dt$ $\Rightarrow \frac{1}{2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)}dx = dt$

	$\Rightarrow \frac{1}{\sin x}dx = dt$ $\therefore I = \int t dt$ $= \frac{t^2}{2} + c$ $= \frac{\left(\log\left(\tan\frac{x}{2}\right)\right)^2}{2} + c$ ans. (ii) We have, $f(x) = \int f'(x)dx$ $f(x) = \int (x+b)dx$ $f(x) = \frac{x^2}{2} + bx + c$ $f(1) = 5$ and $f(2) = 13$ .....(given) $5 = \frac{1}{2} + b + c$ $\frac{9}{2} = b + c$ .....(1) and $13 = 2 + 2b + c$ $11 = 2b + c$ .....(2) solving (1) & (2) $b = \frac{13}{2}$ and $c = -2$ $\therefore f(x) = \frac{x^2}{2} + \frac{13}{x}x - 2$ ans. (iii) $f(x) = x^3 + \frac{1}{x^2} + c$ ans. (iv) $I = \int \frac{e^{x-1}+e^{-1}}{e^x+x^e}dx$ put $e^x + x^e = t$ $e^x + ex^{e-1}dx = dt$ $\Rightarrow e(e^{x-1} + x^{e-1})dx = dt$ $\Rightarrow e^{x-1} + x^{e-1}dx = \frac{dt}{e}$ $\therefore I = \frac{1}{e}\int \frac{dt}{t}$ $= \frac{1}{e}\log \mid t \mid + c$ $I = \frac{1}{e}\log \mid e^x + x^e \mid + c$ ans.
Q.9)	(i) $I = \int \frac{1}{x+\sqrt{x}}dx$ (ii) $I = \int \frac{(x+1)e^x}{\cos^2(xe^x)}dx$ (iii) $I = \int \frac{x^5}{\sqrt{1+x^3}}dx$
Sol.9)	(i) $I = \int \frac{1}{x+\sqrt{x}}dx$ $= \int \frac{1}{\sqrt{x}(\sqrt{x}+1)}dx$ put $\sqrt{x} + 1 = t$ $\frac{1}{2\sqrt{x}}dx = dt$ $\frac{1}{\sqrt{x}}dx = 2dt$ $\therefore I = 2\int \frac{dt}{t}$ $= 2\log \mid \sqrt{x} + 1 \mid + c$ ans.

	(ii)$I = \int \frac{(x+1)e^x}{\cos^2(xe^x)} dx$ put $xe^x = t$ $(xe^x + e^x)dx = dt$ $e^x(x+1)dx = dt$ $\therefore I = \int \frac{dt}{\cos^2 t}$ $= \int sec^2 t dt$ $= \tan t + c$ $= \tan(xe^x) + c$ ans. (iii)$I = \int \frac{x^5}{\sqrt{1+x^3}} dx$ $= \int \frac{x^3 . x^2}{\sqrt{1+x^3}} dt$ put $1 + x^3 = t^2$ ....(1) $3x^2 dx = 2tdt$ $x^2 dx = \frac{2t}{3} dt$ $\therefore I = \frac{2}{3} \int \frac{x^3 . t}{t} dt$ $= \frac{2}{3} \int (t^2 - 1) dt$ ......(from (1)) $= \frac{2}{3} \left(\frac{t^3}{3} - t \right) + c$ $I = \frac{2}{3} \left[\frac{(1+x^3)^{3/2}}{3} - (1+x^3)^{1/2} \right] + c$ ans.
Q.10)	(i) $I = \int \frac{1}{16-9x^2} dx$ (ii)$I = \int \frac{1}{\sqrt{16-9x^2}} dx$ (iii)$I = \int \frac{1}{4+9x^2} dx$ (iv)$I = \int \frac{1}{\sqrt{4+9x^2}} dx$ (v) $I = \int \frac{1}{9x^2-4} dx$
Sol.10)	(i) $I = \int \frac{1}{16-9x^2} dx$ $= \frac{1}{9} \int \frac{1}{\left(\frac{4}{3}\right)^2 - x^2} dx$ $= \frac{1}{9} \times \frac{1}{\frac{2\times 4}{3}} \log \left\| \frac{\frac{4}{3}+x}{\frac{4}{3}-x} \right\| + c$ ......$\int \frac{1}{a^2 x^2} dx = \frac{1}{2a} \log$ $= \frac{1}{24} \log \left\| \frac{4+3x}{4-3x} \right\| + c$ ans. (ii)$I = \int \frac{1}{\sqrt{16-9x^2}} dx$ $= \frac{1}{3} \int \frac{1}{\sqrt{\left(\frac{4}{3}\right)^2 - x^2}} dx$ $= \frac{1}{3} \sin^{-1} \left(\frac{x}{\frac{4}{3}} \right) + c$ ....$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c$ $= \frac{1}{3} \sin^{-1} \left(\frac{3x}{4} \right) + c$ ans. (iii)$I = \int \frac{1}{4+9x^2} dx$ $= \frac{1}{9} \int \frac{1}{\left(\frac{2}{3}\right)^2 + x} dx$

$$= \frac{1}{9} \times \frac{1}{\frac{2}{3}} \tan^{-1}\left(\frac{x}{\frac{2}{3}}\right) + c$$

$$= \frac{1}{6} \tan^{-1}\left(\frac{3x}{2}\right) + c$$ ans.

(iv) $I = \int \frac{1}{\sqrt{4+9x^2}} dx$

$$= \frac{1}{3} \int \frac{1}{\sqrt{\left(\frac{2}{3}\right)^2 + x^2}} dx$$

$$= \frac{1}{3} \log\left| x + \sqrt{\left(\frac{2}{3}\right)^2 + x^2} \right| + c$$ ans.

(v) $I = \int \frac{1}{9x^2-4} dx$

$$= \frac{1}{9} \int \frac{1}{x^2 - \left(\frac{2}{3}\right)^2} dx$$

$$= \frac{1}{9} \times \frac{1}{\frac{2\times2}{3}} \log\left|\frac{x-\frac{2}{3}}{x+\frac{2}{3}}\right| + c$$

$$= \frac{1}{12} \log\left|\frac{3x-2}{3x+2}\right| + c$$ ans.