Integration (Indefinite Integrals)

Q.1)	(i) $I = \int \frac{\cos(2x)-\cos(2\alpha)}{\cos x-\cos\alpha}dx$ (ii)$I = \int \frac{1+\cos(4x)}{\cot x-\tan x}dx$
Sol.1)	(i) $I = \int \frac{\cos(2x)-\cos(2x)}{\cos x-\cos x}dx$ $= \int \frac{(2\cos^2 x-1)-(2\cos^2\alpha-1)}{\cos x-\cos\alpha}dx$ $= 2\int \frac{\cos^2 x-\cos^2\alpha}{\cos x-\cos\alpha}dx$ $= 2\int \frac{(\cos x+\cos\alpha)(\cos x-\cos\alpha)}{\cos x-\cos\alpha}dx$ $= I = 2[\sin x + x\cos\alpha] + c$ ans. (ii)$I = \int \frac{1+\cos(4x)}{\cot x-\tan x}dx$ $= \int \frac{2\cos^2(2x)}{\frac{\cos x}{\sin x}-\frac{\sin x}{\cos x}}dx$ $= \int \frac{2\cos^2(2x)}{\frac{\cos^2 x-\sin^2 x}{\sin x\cos x}}dx$ $= \int \frac{2\sin x.\cos x.\cos^2(2x)}{\cos^2 x-\sin^2 x}dx$ $= \int \frac{\sin(2x).\cos^2(2x)}{\cos(2x)}dx$ $= \int \sin(2x).\cos(2x)dx$ $= \frac{1}{2}\int 2\sin(2x).\cos(2x)dx$ $= \frac{1}{2}\int \sin(4x)dx$ $= \frac{1}{2}\left(-\frac{\cos(4x)}{4}\right) + c$ $I = -\frac{1}{8}\cos(4x) + c$ ans.
→	**Type : Rationalize :** $\int \frac{1}{1\pm\sin x}dx, \int \frac{1}{1\pm\cos x}dx$
Q.2)	(i) $I = \int \frac{1}{1+\sin x}dx$ (ii)$I = \int \frac{\sin x}{1-\sin x}dx$
Sol.2)	(i) $I = \int \frac{1}{1+\sin x}dx$ Rationalize $= \int \frac{1}{1+\sin x} \times \frac{1-\sin x}{1-\sin x}dx$ $= \int \frac{1-\sin x}{\cos^2 x}dx$ Separate $I = \int \frac{1}{\cos^2 x} - \frac{\sin x}{\cos^2 x}dx$ $= \int sec^2 x - \tan x sec x dx$ $I = \tan x - sec x + c$ ans.

	(ii)$I = \int \frac{\sin x}{1-\sin x} dx$ Rationalize = $\int \frac{\sin x(1+\sin x)}{(1-\sin x)(1+\sin x)} dx$ = $\int \frac{\sin x+\sin^2 x}{\cos^2 x} dx$ Separate = $\int \tan x . secx + \tan^2 x dx$ = $\int \tan x . secx + sec^2 x - 1dx$ $I = -secx + \tan x - x + c$ ans.
Q.3)	(i) $I = \int \frac{\cos x-\cos(2x)}{1-\cos x} dx$ (ii)$I = \int \tan^{-1} \sqrt{\frac{1-\cos(2x)}{1+\cos(2x)}} dx$ (iii) $I = \int \tan^{-1}(secx + \tan x)dx$
Sol.3)	(i) $I = \int \frac{\cos x-\cos(2x)}{1-\cos x} dx$ = $\int \frac{\cos x-(2\cos^2 x-1)}{1-\cos x} dx$ = $-\int \frac{2\cos^2 x-\cos x-1}{1-\cos x} dx$ = $-\int \frac{(2\cos x+1)(\cos x-1)}{1-\cos x} dx$ = $-\int \frac{(2\cos x+1)(\cos x-1)}{-(\cos x-1)} dx$ = $\int 2\cos x + 1dx$ = $I = 2\sin x + x + c$ ans. (ii)$I = \int \tan^{-1} \sqrt{\frac{1-\cos(2x)}{1+\cos(2x)}} dx$ = $\int \tan^{-1} \sqrt{\frac{2\sin^2 x}{2\cos^2 x}} dx$ = $\int \tan^{-1}(\tan x)dx$ = $\int xdx$ $I = \frac{x^2}{2} + c$ ans. (iii) $I = \int \tan^{-1}(secx + \tan x)dx$ = $\int \tan^{-1} \left(\frac{1}{\cos x} + \frac{\sin x}{\cos x}\right) dx$ = $\int \tan^{-1} \left(\frac{1+\sin x}{\cos x}\right) dx$ = $\int \tan^{-1} \left[\frac{1+\cos\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)}\right] dx$

	$= \int \tan^{-1}\left(\frac{2\cos^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}{2\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)}\right)dx$ $= \int \tan^{-1}\left(\cot\left(\frac{\pi}{4}-\frac{x}{2}\right)\right)dx$ $= \int \tan^{-1}\left[\tan\left(\frac{\pi}{2}-\left(\frac{\pi}{4}-\frac{x}{2}\right)\right)\right]dx$ $= \int \frac{\pi}{2}-\frac{\pi}{4}+\frac{x}{2}dx$ $= \int \frac{\pi x}{4}+\frac{x}{2}dx$ $= \int \frac{\pi}{4}+\frac{x^2}{2}+c$ ans.
Q.4)	(i) $I = \int \frac{\cos(2x)}{(\cos x+\sin x)^2}dx$ (ii)$I = \int \frac{\cos x-\sin x}{1+\sin(2x)}dx$ (iii) $I = \int \frac{1-\tan x}{1+\tan x}dx$
Sol.4)	(i) $I = \int \frac{\cos(2x)}{(\cos x+\sin x)^2}dx$ $= \int \frac{\cos^2 x-\sin^2 x}{(\cos x+\sin x)^2}dx$ $= \int \frac{(\cos x+\sin x)(\cos x-\sin x)}{(\cos x+\sin x)^2}dx$ $= \int \frac{\cos x-\sin x}{\cos x+\sin x}dx$ put cos x sin x = t (–sin x + cos x) dx = dt $= \int \frac{dt}{t}$ $= \log \mid t \mid +c$ $= I = \log \mid \cos x + \sin x \mid +c$ ans. (ii)$I = \int \frac{\cos x-\sin x}{1+\sin(2x)}dx$ $= \int \frac{\cos x-\sin x}{\sin^2 x+\cos^2 x+2\sin x\cos x}dx$ $= \int \frac{\cos x-\sin x}{(\sin x+\cos x)^2}dx$ put sin *x* + cos *x* = *t* (cos x – sin x) dx = dt $= \int \frac{dt}{t^2}$ $= \int -\frac{1}{t}+c$ $= I = -\frac{1}{\sin x+\cos x}+2$ ans. (iii) $I = \int \frac{1-\tan x}{1+\tan x}dx$ $= \int \frac{\frac{1-\sin x}{\cos x}}{\frac{1+\sin x}{\cos x}}dx$

	$= \int \frac{\cos x - \sin x}{\cos x + \sin x} dx$ put cos x + sin x = t (−sin x + cos x) dx = dt $\therefore I = \int \frac{dt}{t}$ $= I = \log \mid \sin x + \cos x \mid + c$ ans.
Q.5)	(i) $I = \int \frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x.\cos^2 x} dx$ (ii) $I = \int \frac{1}{\sqrt{\sin^3 x \sin(x+\alpha)}} dx$
Sol.5)	(i) $I = \int \frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x.\cos^2 x} dx$ $= \int \frac{(\sin^4 x)^2 - (\cos^4 x)^2}{(\sin^2 x + \cos^2 x)^2 - 2\sin^2 x.\cos^2 x} dx$ ......{1 = sin² x + cos x} $= \int \frac{(\sin^4 x + \cos^4 x)(\sin^4 x - \cos^4 x)}{\sin^4 x + \cos^4 x + 2\sin^2 x.\cos^2 x - 2\sin^2 x.\cos^2 x} dx$ $= \int \frac{(\sin^4 x + \cos^4 x)(\sin^4 x - \cos^4 x)}{\sin^4 x + \cos^4 x} dx$ $= \int (\sin^2 x + \cos x)(\sin^2 x - \cos^2 x) dx$ $= \int (1)[-\cos(2\text{x})] dx$ ......{cos(2x) = cos² x – sin² x} $= -\int \cos(2\text{x}) dx$ $= \frac{-\sin(2\text{x})}{2} + c$ ans. (ii) $I = \int \frac{1}{\sqrt{\sin^3 x \sin(x+\alpha)}} dx$ $= \int \frac{1}{\sqrt{\sin^3 x.(\sin x \cos\alpha + \cos x \sin\alpha)}} dx$ take sin x common $= \int \frac{1}{\sqrt{\sin^4 x.(\cos\alpha + \cot x \sin\alpha)}} dx$ $= \int \frac{1}{\sqrt{\sin^2 x.(\cos\alpha + \cot x \sin\alpha)}} dx$ $= \int \frac{cosec^2 x}{\sqrt{\cos\alpha + \cot x \sin\alpha}} dx$ $= \int \frac{cosec^2 x}{\sqrt{(\cos\alpha + \cot x \sin\alpha)}} dx$ put $\cos\alpha + \cot x \sin\alpha = t$ $\therefore -cosec^2 x. \sin\alpha dx = dt$ $cosec^2 x dx = \frac{-dt}{\sin\alpha}$ $\therefore I = -\frac{1}{\sin\alpha} \int \frac{dt}{\sqrt{t}}$ $= \frac{-1}{\sin\alpha} \times 2\sqrt{t} + c$ $I = -\frac{1}{\sin\alpha} 2\sqrt{\cos\alpha + \cot x \sin\alpha} + c$ ans.
Q.6)	(i) $I = \int \frac{\sin(2\text{x})}{(a + b\cos x)^2} dx$ (ii) $I = \int \frac{1}{\sqrt{1 - \sin x}} dx$
Sol.6)	(i) $I = \int \frac{\sin(2\text{x})}{(a + b\cos x)^2} dx$

$= 2\int \frac{\sin x . \cos x}{(a+b\cos x)^2} dx$

put a + b cos x = t

$\therefore$ –b sin x dx = dt

$\sin x dx = \frac{-dt}{5}$

$I = \frac{-2}{b} \int \frac{\cos x}{t^2} dt$

$= \frac{-2}{b^2} \int \frac{1}{t^2} . \left(\frac{t-a}{b}\right) dt$

$= \frac{-2}{b^2} \int \frac{t-a}{t^2} dt$

$= \frac{-2}{b^2} \int \frac{1}{t} - \frac{a}{t^2} dt$

$= \frac{-2}{b^2} \left[\log | t | + \frac{a}{t}\right] + c$

$I = \frac{-2}{b^2} \left[\log | a + b\cos x | + \frac{a}{a+b\cos x}\right] + c$ ans.

(ii) $I = \int \frac{1}{\sqrt{1-\sin x}} dx$

$= \int \frac{1}{\sqrt{1-\cos\left(\frac{\pi}{2}-x\right)}} dx$

$= \int \frac{1}{\sqrt{2\sin^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}} dx$

$= \frac{1}{\sqrt{2}} \int cosec \left(\frac{\pi}{4} - \frac{x}{2}\right) dx$

$= \frac{1}{\sqrt{2}} . \log \left| cosec \left(\frac{\pi}{4} - \frac{x}{2}\right) - \cot \left(\frac{\pi}{4} - \frac{x}{2}\right) \right| \times (-2) + c$

$= -\sqrt{2} . \log \left| cosec \left(\frac{\pi}{4} - \frac{x}{2}\right) - \cot \left(\frac{\pi}{4} - \frac{x}{2}\right) \right| + c$ ans

→ **Type :** $\int linear\sqrt{Linear} dx, \frac{linear}{\sqrt{Linear}} dx, \int \frac{linear}{\sqrt{(Linear)^n}} dx$

Put Linear = *t* or *t*^{2} (Or) make adjustments

Q.7) (i) $I = \int x\sqrt{x+2} dx$ (ii) $I = \int (7x - 2)\sqrt{3x + 2} dx$

(iii) $I = \int \frac{2x+3}{(x-1^2)} dx$

Sol.7) (i) $I = \int x\sqrt{x+2} dx$

put $x + 2 = t^2$

$dx = 2t dt$

$\therefore I = 2\int x . \sqrt{t^2} t dt$

$= 2\int (t^2 - 2) . t . t dt$

$= 2\int t^4 - 2t^2 dt$

$= 2\left[\frac{t^5}{5} - \frac{2t^3}{5}\right] + c$

replacing t by $(x+2)^{\frac{1}{2}}$

	$= \ 2\left[\frac{(x+2)^{\frac{5}{2}}}{5} - 2\frac{(x+2)^{3/2}}{3}\right] + c$ ans. Alternate Method: (adjustment) $I = \int x\sqrt{x+2}dx$ $= \int (x+2-2)\sqrt{x+2}dx$ $= \int (x+2)^{\frac{3}{2}} - 2\sqrt{x+2}dx$ $= \frac{2}{5}(x+2)^{\frac{5}{2}} - 2 \times \frac{2}{3}(x+2)^{\frac{3}{2}} + c$ ans. (ii)$I = \int (7\text{x}-2)\sqrt{3\text{x}+2}dx$ put $3\text{x}+2 = t^2$ $3\text{dx} = 2\text{t}dt$ $dx = \frac{2}{3}tdt$ $I = \frac{2}{3}\int (7\text{x}-2)\sqrt{t^2}.tdt$ $= \frac{2}{3}\int \left[7\left(\frac{t^2-2}{3}\right) - 2\right]t.tdt$ $= \frac{2}{3}\int \frac{(7\text{t}^2-14-6)}{3}t^2dt$ $= \frac{2}{9}\int 7\text{t}^4 - 20\text{t}^2dt$ $= \frac{2}{9}\left[\frac{7\text{t}^5}{5} - \frac{20\text{t}^3}{3}\right] + c$ replacing t by $(3\text{x}+2)^{\frac{1}{2}}$ $\therefore I = \frac{2}{9}\left[\frac{7}{5}(3\text{x}+2)^{\frac{5}{2}} - \frac{20}{3}(3\text{x}+2)^{\frac{3}{2}}\right] + c$ ans. (iii)$I = \int \frac{2\text{x}+3}{(x-1^2)}dx$ put $x-1 = t$ $dx = dt$ $\therefore I = \int \frac{2\text{x}+3}{t^2}dt$ $= \int \frac{2(t+1)+3}{t^2}dt$ $= \int \frac{2\text{t}+5}{t^2}dt$ $= \int \frac{2}{t} + \frac{5}{t^2}dt$ $= 2\log \mid t \mid - \frac{5}{t} + c$ $I = 2\log \mid x-1 \mid - \frac{5}{x-1} + c$ ans.
Q.8)	(i) $I = \int \frac{1}{\sqrt{x+a}+\sqrt{x+b}}dx$
Sol.8)	(i) $I = \int \frac{1}{\sqrt{x+a}+\sqrt{x+b}}dx$

	Rationalize $I = \int \dfrac{\sqrt{x+a} - \sqrt{x+b}}{(x+a)-(x+b)}\,dx$ $= \int \frac{\sqrt{x+a}-\sqrt{x+b}}{a-b}\,dx$ $= \frac{1}{a-b}\int \sqrt{x+a} - \sqrt{x+b}\,dx$ $= \frac{1}{a-b}\left[\frac{2}{3}(x+a)^{\frac{3}{2}} - \frac{2}{3}(x+b)^{\frac{3}{2}}\right] + c$ ans.
Q.9)	(i) $I = \int \frac{(x^4-x)^{\frac{1}{4}}}{x^5}\,dx$ (ii) $I = \int \frac{1}{x^2(x^4+1)^{\frac{3}{4}}}\,dx$
Sol.9)	(i) $I = \int \frac{(x^4-x)^{\frac{1}{4}}}{x^5}\,dx$ take x^4 common $= \int \frac{x\left(1-\frac{x}{x^4}\right)^{\frac{1}{4}}}{x^5}\,dx$ $= \int \frac{\left(1-\frac{1}{x^3}\right)^{\frac{1}{4}}}{x^4}\,dx$ put $1 - \frac{1}{x^3} = t$ $\frac{3}{x^4}dx = dt \Rightarrow \frac{dx}{x^4} = \frac{dt}{3}$ $\therefore I = \frac{1}{3}\int t^{\frac{1}{4}}dt$ $= \frac{1}{3}.\frac{4}{5}t^{\frac{5}{4}} + c$ $= I = \frac{4}{15}\left(1-\frac{1}{x^3}\right)^{\frac{5}{4}} + c$ ans. (ii) $I = \int \frac{1}{x^2(x^4+1)^{\frac{3}{4}}}\,dx$ take x^4 common $= \int \frac{1}{x^2.x^3\left(1+\frac{1}{x^4}\right)^{\frac{3}{4}}}\,dx$ $= \int \frac{1}{x^5\left(1+\frac{1}{x^4}\right)^{\frac{3}{4}}}\,dx$ Put $\frac{1+1}{x^4} = t$ $\frac{-4}{x^5}dx \Rightarrow \frac{dx}{x^5} = \frac{-dt}{5}$ $\therefore I = \frac{-1}{5}\int \frac{1}{t^{\frac{3}{4}}}dt$ $= \frac{-1}{5}\int t^{\frac{-3}{4}}dt$ $= \frac{-1}{5}\left(t^{\frac{1}{4}} \times 4\right) + c$ $= \frac{-4}{5}\left(1+\frac{1}{x^4}\right)^{\frac{1}{4}} + c$ ans.
Q.10)	(i) $I = \int \frac{1}{x\sqrt{ax-x^2}}\,dx$ (ii) $I = \int \frac{1}{x(x^n+1)}\,dx$

Sol.10)	(i) $I = \int \frac{1}{x\sqrt{ax-x^2}}dx$ take x^2 common = $\int \frac{1}{x.x\sqrt{\frac{a}{x}-1}}dx$ = $\int \frac{1}{x^2\sqrt{\frac{a}{x}-1}}dt$ put $\frac{a}{x} - 1 = t$ $\frac{-a}{x^2}dx = dt \Rightarrow \frac{1}{x^2}dx = -\frac{dt}{a}$ $I = \frac{-1}{a}\int \frac{dt}{\sqrt{t}}$ = $\frac{-1}{a} \times 2\sqrt{t} + c$ = $I = \frac{-2}{a}\sqrt{\frac{a}{x} - 1} + c$ ans. (ii)$I = \int \frac{1}{x(x^n+1)}dx$ take x^n common = $\int \frac{1}{x^{n+1}\left(1+\frac{1}{x^n}\right)}dx$ put $\frac{1+1}{x^n} = t$ $\frac{-n}{x^{n+1}}dx = dt$ $\Rightarrow \frac{1}{x^{n+1}}dx = -\frac{dt}{n}$ $\therefore I = \frac{-1}{n}\int \frac{1}{t}dt$ = $\frac{-1}{n}\log \mid t \mid +c$ = $\frac{-1}{n}\log\left\|1 + \frac{1}{x^n}\right\| + c$ ans.