<u>Integration</u> (Indefinite Integrals)

Q.1)	$I = \int x\cos^3(x^2)\sin(x^2)dx$
Sol.1)	$I = \int x\cos^3(x^2)\sin(x^2)dx$ put $x^2 = t$ $2x\ dx = dt \Rightarrow x\ dx = \frac{1}{2}dt$ $\therefore I = \frac{1}{2}\int \cos^3 t.\sin t dt$ put cos $t = z$ sin $t\ dt = -dz$ $\therefore I = -\frac{1}{2}\int z^3 dx$ = $\frac{-1}{2}\frac{z^4}{4} + c$ replacing 'z' = $\frac{-1}{8}\cos^4 t + c$ replacing t = $\frac{-1}{8}\cos^4(x^2) + c$ ans.
Q.2)	(i) $I = \int \frac{1}{\sin^3 x.\cos x}dx$
Sol.2)	(i) $I = \int \frac{1}{\sin^3 x.\cos x}dx$ Divide N & D by $\cos^4 x$ $I = \int \frac{sec^4 x}{\tan x}dx$ = $\frac{\int sec^2 x.sec^2 x}{\tan x}dx$ = $\frac{\int (1+\tan^2 x).sec^2 x}{\tan x}dx$ put tan $x = t$ $\therefore$ $\sec^2 x\ dx = dt$ $I = \int \frac{(1+t^2)}{t}dt$ = $\int \frac{1}{t} + t dt$ = $\log\mid t \mid = \frac{t^2}{2} + c$ $I = \log\mid \tan x \mid + \frac{1}{2}\tan^2 x + c$ ans.
Q.3)	(i) $I = \int \frac{1}{\sin^4 x.\cos^2 x}dx$
Sol.3)	(i) $I = \int \frac{1}{\sin^4 x.\cos^2 x}dx$

	Divide N & D by $\cos^6 x$ $= \int \frac{sec^6x}{\tan^2x}dx$ $= \int \frac{sec^4x.sec^2x}{\tan^2x}dx$ $= \int \frac{(1+\tan^2x)^2.sec^2x}{\tan^2x}dx$ put $\tan x = t$ $\sec^2 x\, dx = dt$ $\therefore I = \int \frac{(1+t^2)^2}{t^2}dt$ $= \int \frac{1+t^4+2t^2}{t^2}dt$ $= \int \frac{1}{t^2} + t^2 + 2dt$ $= \frac{-1}{t} + \frac{t^3}{3} + 2t + c$ $\therefore I = -\frac{1}{\tan x} + \frac{\tan^3 x}{3} + 2\tan x + c$ ans.
Q.4)	$I = \int \frac{\cos^9 x}{\sin x}dx$
Sol.4)	$I = \int \frac{\cos^9 x}{\sin x}dx$ $= \int \frac{\cos^8 x.\cos x}{\sin x}dx$ $= \int \frac{(\cos^2 x)^4.\cos x}{\sin x}dx$ $= \int \frac{(1-\sin^2 x)^4 \cos x dx}{\sin x}$ put $\sin x = t$ $\therefore \cos x\, dx = dt$ $= \int \frac{(1-t^2)^4}{t}dt$ $= \int \frac{(1+t^4-2t^2)^2}{t}dt$ $= \int \frac{1+t^3+4t^4+2t^4-4t^6-4t^2}{t}$ $= \int \frac{t^8-4t^6+4t^4-4t^2+1}{t}dt$ $= \int t^7 - 4t^5 + 4t^3 - 4t + \frac{1}{t}dt$ $= \int \frac{t^8}{8} - \frac{4t^6}{6} + \frac{4t^4}{4} - \frac{4t^2}{2} + \log\mid t \mid +c$ $= \int \frac{\sin^8 x}{8} - \frac{2}{3}\sin^6 x + \sin^4 x - 2\sin^2 x + \log\mid \sin x \mid +c$
→	**<u>Sin *x* and Cos *x* in multiplication with different Angles</u> :-**
Q.5)	(i) $I = \int \sin(3x)\cos(2x)dx$
Sol.5)	(i) $I = \int \sin(3x)\cos(2x)dx$

	$= \frac{1}{2}\int 2\sin(3x).\cos(2x)dx$ $= \frac{1}{2}\int \sin(5x)+\sin(x)dx$ $I = \frac{1}{2}\left[\frac{-\cos(5x)}{5} - \cos x\right] + c$ ans.
Q.6)	(i) $I = \int \sin(2x).\sin(4x).\sin(6x)dx$ (ii) $I = \int \frac{\sin(4x)}{\sin x}dx$
Sol.6)	(i) $I = \int \sin(2x).\sin(4x).\sin(6x)dx$ $= \frac{1}{2}\int [2\sin(2x).\sin(4x)].\sin(6x)dx$ $= \frac{1}{2}\int [\cos(2x)-\cos(6x)].\sin(6x)dx$ $= \frac{1}{2}\int \sin(6x).\cos(2x)-\sin(6x).\cos(6x)dx$ $= \frac{1}{4}\int 2\sin(6x)\cos(2x)-2\sin(6x).\cos(6x)dx$ $= \frac{1}{4}\int \sin(8x)+\sin(4x)-\sin(12x)dx$ $= \frac{1}{4}\left[\frac{-\cos(8x)}{8} - \frac{\cos(4x)}{4} + \frac{\sin(12x)}{12}\right] + c$ ans. iii) $I = \int \frac{\sin(4x)}{\sin x}dx$ $= 2\int \frac{\sin(2x).\cos(2x)}{\sin x}dx$ $= 4\int \frac{\sin x.\cos x.\cos(2x)}{\sin x}dx$ $= \int 2\cos x.\cos(2x)dx$ $= 2\int \cos(3x)+\cos x dx$ ……{2 cos A cos B = cos (A + B) + cos(A – B)} $= 2\left[\frac{\sin(3x)}{3} + \sin x\right] + c$ ans.
Q.7)	(i) $I = \int \tan x.sec^4x\ dx$ (ii) $I = \int \tan^3x.sec^3x\ dx$ (iii) $I = \int sec^nx.\tan x\ dx$
Sol.7)	(i) $I = \int \tan x.sec^4x\ dx$ $= \int \tan x.(1+\tan^2x).sec^2xdx$ put tan *x* = *t* ∴ $\sec^2 x\ dx = dt$ $I = \int t(1+t^2)dt$ $= \int t+t^3dt$ $= \frac{t^2}{2} + \frac{t^4}{4} + c$ $I = \frac{\tan^2x}{2} + \frac{\tan^4x}{4} + c$ ans. (ii) $I = \int \tan^3x.sec^3xdx$ $= \int \tan^2x.sec^2x.\tan x.secxdx$

$= \int (sec^2x - 1).sec^2x - \tan x.secxdx$

put sec $x = t$

sec x tan x $dx = dt$

$I = \int (t^2 - 1)t^2dt$

$= \int t^4 - t^2dt$

$= \frac{t^5}{5} - \frac{t^3}{3} + c$

$I = \frac{sec^5x}{5} - \frac{sec^3}{3} + c$ ans.

(iii)$I = \int sec^nx.\tan xdx$

$= \int \sin^{n-1}x.\tan x.secxdx$

put sec $x = t$

sec x tan x $dx = dt$

$I = \int t^{n-1}dt$

$= \frac{t^n}{n} + c$

$I = \frac{sec^nx}{n} + c$ ans.

→ <u>QNS Based On Sin(A ± B) and Cos(A ± B)</u> :-

Q.8) (i) $I = \int \frac{\sin(x-a)}{\sin x}dx$ (ii)$I = \int \frac{\sin x}{\sin(x+a)}dx$ (iii)$I = \int \frac{\sin(x+a)}{\sin(x+b)}dx$

Sol.8) (i) $I = \int \frac{\sin(x-a)}{\sin x}dx$

$= \int \frac{\sin x.\cos a - \cos x.\sin a}{\sin x}dx$

Separate

$= \int \cos a - \cot x.\sin adx$

$I = x\cos a + \log|\sin x|.\sin a + c$

(ii) $I = \int \frac{\sin x}{\sin(x+a)}dx$

$= \int \frac{\sin(x+a-a)}{\sin(x+a)}dx$

$= \int \frac{\sin(x+a).\cos a}{\sin(x+a)} - \frac{\cos(x+a)\sin a}{\sin(x+a)}dx$

$= \int \cos a - \cot(x+a)\sin a \ dx$

$I = x\cos a - \log|\sin(x+a)|\sin a + c$ ans.

(iii)$I = \int \frac{\sin(x+a)}{\sin(x+b)}dx$

$= \int \frac{\sin(x+a+b-b)}{\sin(x+b)}dx$

$= \int \frac{\sin[(x+b)+(a-b)]}{\sin(x+b)}dx$

$= \int \frac{\sin(x+b)\cos(a-b)}{\sin(x+b)} + \frac{\cos(x+b)\sin(a-b)}{\sin(x+b)}dx$

	$= \int \cos(a-b) + \cot(x+b)\sin(a-b)dx$ $I = x\cos(a-b) + \log\mid\sin(x+b)\mid\sin(a-b) + c$ ans.
Q.9)	(i) $I = \int \frac{\sin(2x)}{\sin(5x)\sin(3x)}dx$
Sol.9)	(i) $I = \int \frac{\sin(2x)}{\sin(5x)\sin(3x)}dx$ $= \int \frac{\sin(5x-3x)}{\sin(5x).\sin(3x)}dx$ $= \int \frac{\sin(3x)\cos(3x)-\cos(5x).\sin(3x)}{\sin(5x).\sin(3x)}dx$ Separate $I = \int \cot(3x) - \cot(5x)dx$ $= \frac{1}{3}\log\mid\sin(3x)\mid - \frac{1}{5}\log\mid\sin(5x)\mid + c$ ans.
Q.10)	(i) $I = \int \frac{1}{\sin(x-a)\sin(x-b)}dx$ (ii) $I = \int \frac{1}{\cos(x+a)\cos(x+b)}dx$ (iii) $I = \int \frac{1}{\sin(x+b)\cos(x-b)}dx$
Sol.10)	(i) $I = \int \frac{1}{\sin(x-a)\sin(x-b)}dx$ $= \frac{1}{\sin(a-b)}\int \frac{\sin(a-b)}{\sin(x-a)\sin(x-b)}dx$ $= \frac{1}{\sin(a-b)}\int \frac{\sin(a-b+x-x)}{\sin(x-a)\sin(x-b)}dx$ $= \frac{1}{\sin(a-b)}\int \frac{\sin[(x-b)-(x-a)]}{\sin(x-a)\sin(x-b)}dx$ $= \frac{1}{\sin(a-b)}\int \frac{\sin(x-b)\cos(x-a)-\cos(x-a)\sin(x-b)}{\sin(x-a)\sin(x-b)}dx$ Separate $= \frac{1}{\sin(a-b)}\int \cot(x-a) - \cot(x-b)dx$ $= \frac{1}{\sin(a-b)}\int [\log\mid\sin(x-a)\mid - \log\mid\sin(x-b)\mid] + c$ $I = \frac{1}{\sin(a-b)}\log\left\|\frac{\sin(x-a)}{\sin(x-b)}\right\| + c$ ans. (ii) $I = \int \frac{1}{\cos(x+a)\cos(x+b)}dx$ $= \frac{1}{\sin(a-b)}\int \frac{\sin(a-b)}{\cos(x+a).\cos(x+b)}dx$ $= \frac{1}{\sin(a-b)}\int \frac{\sin(a-b+x-x)}{\cos(x+a).\cos(x+b)}dx$ $= \frac{1}{\sin(a-b)}\int \frac{\sin[(x+a)-(x+b)]}{\cos(x+a).\cos(x+b)}dx$ $= \frac{1}{\sin(a-b)}\int \frac{\sin(x+a)\cos(x+b)-\cos(x+a)\sin(x+b)}{\cos(x+a).\cos(x+b)}dx$ Separate: $= \frac{1}{\sin(a-b)}\int \tan(x+a) - \tan(x+b)dx$

	$= \frac{1}{\sin(a-b)}[\log \mid sec(x+a) \mid -\log \mid sec(x+b) \mid] + c$ $I = \frac{1}{\sin(a-b)} \log\left\|\frac{sec(x+a)}{sec(x+b)}\right\| + c$ ans. (iii)$I = \int \frac{1}{\sin(x-a)\cos(x-b)} dx$ $= \frac{1}{\cos(a-b)} \int \frac{\cos(a-b)}{\sin(x-a).\cos(x-b)} dx$ $= \frac{1}{\cos(a-b)} \int \frac{\cos(a-b+x-x)}{\sin(x-a).\cos(x-b)} dx$ $= \frac{1}{\cos(a-b)} \int \frac{\cos[(x-b)-(x-a)]}{\sin(x-a).\cos(x-b)} dx$ $= \frac{1}{\cos(a-b)} \int \frac{\cos(x-b)\cos(x-a)+\sin(x-b)\sin(x-a)}{\sin(x-a).\cos(x-b)} dx$ Separate $= \frac{1}{\cos(a-b)} \int \cot(x-a) + \tan(x-b) dx$ $I = \frac{1}{\cos(a-b)}[\log \mid \sin(x-a) \mid +\log \mid sec(x-b) \mid] + c$ ans.