Determinants
Class 12th

Q.1)	Find the value of x if the area of Δ is 35 square units with vertices (x, 4), (2, –6) and (5, 4).
Sol.1)	Let vertices are $A(x,4)\,,B(2,-6)\ and\ C(5,4)$ Area of ΔABC = $\frac{1}{2}\begin{vmatrix} x & 4 & 1 \\ 2 & -6 & 1 \\ 5 & 4 & 1 \end{vmatrix}$ 35 = $\frac{1}{2}\ \lvert x(-10)-4(-3)+1(38)\rvert$ ⇒ 35 = $\frac{1}{2}\ \lvert -10x+12+38\rvert$ ⇒ 70 = $\lvert -10x+50\rvert$ $70=-10x+50$ | $-70=-10x+50$ $10x=-20$ | $10x=120$ $x=-2$ | $x=12$ ∴ $x=-2\ ,\ x=12$ ans.
Q.3)	Find the value of x so that matrix A = $\begin{bmatrix} (x-1) & 1 & 1 \\ 1 & (x-1) & 1 \\ 1 & 1 & (x-1) \end{bmatrix}$ is singular/ Non-Invertible.
Sol.3)	Since matrix A is singular ∴ |A| = 0 $\begin{vmatrix} x-1 & 1 & 1 \\ 1 & x-1 & 1 \\ 1 & 1 & x-1 \end{vmatrix}=0$ ⇒ (x – 1) [(x – 1)^2 – 1] – 1[x – 1 – 1] + 1[1 – x + 1] = 0 ⇒ (x – 1) (x^2 – 2x) – 1(x – 2) + (2 – x) = 0 ⇒ x^3 – 2x^2 – x^2 + 2x – x + 2 + 2 – x = 0 ⇒ x^3 – 3x^2 + 4 = 0 By trial method $(x+1)\,(x-2)\,(x+1)=0$ ⇒ $x=-1, x=2$ ans.
Q.4)	(a) Evaluate the determinant Δ = $\begin{vmatrix} 1 & sin\,\theta & 1 \\ -sin\,\theta & 1 & sin\,\theta \\ -1 & -sin\,\theta & 1 \end{vmatrix}$. Also prove $2\ \le\ \Delta \le\ 4$. (b) Prove that Δ = $\begin{vmatrix} x & sin\,\theta & cos\,\theta \\ -sin\,\theta & -x & 1 \\ cos\,\theta & 1 & x \end{vmatrix}$ is independent of θ.
Sol.4)	(a) we have, Δ = $\begin{vmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{vmatrix}$ ⇒ $\Delta = 1(1+sin^2\theta) - sin\,\theta(-sin\,\theta + sin\,\theta) + 1(sin^2\theta + 1)$ ⇒ $\Delta = 1 + sin^2\theta + 0 + sin^2\theta + 1$ ⇒ $\Delta = 2 + 2\,sin^2\theta$ Now , we know $-1 \le \sin\theta \le 1$ ⇒ $0 \le \sin^2\theta \le 1$ ⇒ $0 \le 2\sin^2\theta \le 2$ (multiply by 2) ⇒ $2 \le 2 + 2\sin^2\theta \le 4$ (adding 2) ⇒ $2 \le \Delta \le 4$ (proved) (b) $\Delta = x(-x^2-1) - sin\,\theta(-x\,sin\,\theta - cos\,\theta) + cos\,\theta(-sin\,\theta + x\,cos\,\theta)$ $\Delta = -x^3 - x + xsin^2\theta + sin\,\theta\,cos\,\theta - sin\,\theta\,cos\,\theta + x\,cos^2\theta$

	$\Delta = -x^3 - x + x(sin^2\theta + cos^2\theta)$ $\Delta = -x^3 - x + x(1)$ $\Delta = -x^3$ *which is independent* of θ.

Short Questions

Q.5) Order 3×3 , $|A| = 5$. Find $|\text{Adj } A| =?$

Sol.5) We have $n = 3$, $|A| = 5$

and $|Adj\ A| = |A|^{n-1}$

$= (5)^{3-1} = 25$ ans.

Q.6) Order 3×3 , $|\text{Adj } A| = 81$ find $|A|$ =?

Sol.6) We have n = 3 , $|\text{Adj } A| = 81$

$\Rightarrow |\text{Adj } A| = |A|^{n-1}$

$\Rightarrow 81 = |A|^2$

$\Rightarrow |A| = \pm 9$ ans.

Q.7) Order 3×3 ; $|A| = 3$ find $|4A|$ = ?

Sol.7) We have n = 3 , $|A| = 3$

$|4A| = 4^3 |A|$ .....{∵ $|kA| = k^n |A|$}

$= 64 \times 3$

= 192 ans.

Q.8) *Order* 3×3 ; $|A| = 5$ *find* $|2Adj\ A| =?$

Sol.8) $|2Adj\ A| = 2^3|Adj\ A| = 2^3|A|^{3-1}$

$= 8\ (5)^2 = 200$ *ans.*

Q.9) Order 4×4 ; $|3\text{ Adj } A| = 243$ Find $|A|$ =?

Sol.9) We have $|3\text{ Adj } A| = 3^4|\text{Adj } A|$

$243 = 3^4\ |A|^{4-1}$

$243 = 81\ |A|^3$

$|A|^3 = 3$

$|A| = (3)^{1/3}$ ans.

Q.10) Order 4×4 ; $|A| = 5$ find $|A'|$ =?

Sol.10) We know $|A'| = |A|$

$\Rightarrow |A'| = 5$ ans.