Determinants
Class 12th

Q.1)	Show that $\begin{vmatrix} 1 & 1 & 1 \\ mc_1 & m+1c_1 & m+2c_1 \\ mc_2 & m+1c_2 & m+2c_2 \end{vmatrix} = 1$
Sol.1)	We have $\begin{vmatrix} 1 & 1 & 1 \\ mc_1 & m+1c_1 & m+2c_1 \\ mc_2 & m+1c_2 & m+2c_2 \end{vmatrix}$ = $\begin{vmatrix} 1 & 1 & 1 \\ m & m+1 & m+2 \\ \frac{m(m-1)}{2} & \frac{(m+1)m}{m} & \frac{(m+2)(m+1)}{2} \end{vmatrix}$ ..... $\left\{ \because \begin{matrix} nc_1 = n \\ nc_2 = \frac{n(n-1)}{2} \end{matrix} \right\}$ taking $\left(\frac{1}{2}\right)$ common from R_3 = $\frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ m & m+1 & m+2 \\ m^2-m & m^2+m & m^2+3\text{m}+2 \end{vmatrix}$ $c_2 \to c_2 - c_1 \text{and} c_3 \to c_3 - c_2$ = $\frac{1}{2}\begin{vmatrix} 1 & 0 & 0 \\ m & 1 & 2 \\ m^2-m & 2\text{m} & 4\text{m}+2 \end{vmatrix}$ expanding along R_1 = $\frac{1}{2}[4\text{m}+2-4\text{m}]$ = $\frac{2}{2} = 1$ = RHS ans.
Q.2)	Show $\begin{vmatrix} (b+c)^2 & ba & ca \\ ab & (c+a)^2 & cb \\ ac & bc & (a+b)^2 \end{vmatrix} = 2\text{abc}(a+b+c)^3$
Sol.2)	We have $\begin{vmatrix} (b+c)^2 & ba & ca \\ ab & (c+a)^2 & cb \\ ac & bc & (a+b)^2 \end{vmatrix}$ = $R_1 \to aR_1, R_2 \to bR_2, R_3 \to cR_3$ = $\frac{1}{abc}\begin{vmatrix} a(b+c)^2 & ba^2 & ca^2 \\ ab^2 & b(c+a)^2 & cb^2 \\ ac^2 & bc^2 & c(a+b)^2 \end{vmatrix}$ taking a , b , c common from $c_1, c_2 \text{and} c_3$ resp. = $\frac{1}{abc}\begin{vmatrix} a(b+c)^2 & ba^2 & ca^2 \\ ab^2 & b(c+a)^2 & cb^2 \\ ac^2 & bc^2 & c(a+b)^2 \end{vmatrix}$ $c_1 \to c_1 - c_3 \text{and} c_2 \to c_2 - c_3$ = $\begin{vmatrix} (b+c+a)(b+c-a) & 0 & a^2 \\ 0 & (c+a+b)(c-a-b) & b^2 \\ (c+a+b)(c-a-b) & (c+a+b)(c-a-b) & (a+b)^2 \end{vmatrix}$ taking $(a + b + c)$ common from C_1 & C_2 both

	$= (a+b+c)^2 \begin{vmatrix} b+c-a & 0 & a^2 \\ 0 & c+a-b & b^2 \\ c-a-b & c-a-b & (a+b)^2 \end{vmatrix}$ $R_3 \to R_3(R_1+R_2)$ $= (a+b+c)^2 \begin{vmatrix} b+c-a & 0 & a^2 \\ 0 & c+a-b & b^2 \\ -2\text{b} & -2\text{a} & 2\text{ab} \end{vmatrix}$ $c_1 \to ac_1 \text{and} c_2 \to bc_2$ $= \frac{(a+b+c)^2}{ab} \begin{vmatrix} ab+ac-a^2 & 0 & a^2 \\ 0 & bc+ab-b^2 & b^2 \\ -2\text{ab} & -2\text{ab} & 2\text{ab} \end{vmatrix}$ $c_1 \to c_1+c_3 \text{and} c_2 \to c_2+c_3$ $= \frac{(a+b+c)^2}{ab} \begin{vmatrix} ab+ac & a^2 & a^2 \\ b^2 & bc+ab & b^2 \\ 0 & 0 & 2\text{ab} \end{vmatrix}$ taking a , b and 2ab common from R_1 , R_2 and R_3 resp. $= \frac{(a+b+c)^2}{ab}.ab(2\text{ab}) \begin{vmatrix} b+c & a & a \\ b & c+a & b \\ 0 & 0 & 1 \end{vmatrix}$ expanding $= 2\text{ab}(a+b+c)^2[(b+c)(c+a)-a(b)+a(0)]$ $= 2\text{ab}(a+b+c)^2(bc+ab+c^2+ac-ab)$ $= 2\text{ab}(a+b+c)^2.c(b+c+a)$ $= 2\text{abc}(a+b+c)^3$ = RHS ans.
Q.3)	Show that $\begin{vmatrix} 1+a^2-b^2 & 2\text{ab} & -2\text{b} \\ 2\text{ab} & 1-a^2+b^2 & 2\text{a} \\ 2\text{b} & -2\text{a} & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$
Sol.3)	We have $\begin{vmatrix} 1+a^2-b^2 & 2\text{ab} & -2\text{b} \\ 2\text{ab} & 1-a^2+b^2 & 2\text{a} \\ 2\text{b} & -2\text{a} & 1-a^2-b^2 \end{vmatrix}$ Main step $c_1 \to c_1 - bc_3 \text{and} c_2 \to ac_3$ $= \begin{vmatrix} 1+a^2-b^2+2\text{b}^2 & 2\text{ab}-2\text{ab} & -2\text{b} \\ 2\text{ab}-2\text{ab} & 1-a^2+b^2+2\text{a}^2 & 2\text{a} \\ 2\text{b}-b+a^2b+b^3 & -2\text{a}+a-a^3-ab^2 & 1-a^2-b^2 \end{vmatrix}$ $= \begin{vmatrix} 1+a^2+b^2 & 0 & -2\text{b} \\ 0 & 1+a^2+b^2 & 2\text{a} \\ b(1+a^2+b^2) & -a(1+a^2+b^2) & 1-a^2-b^2 \end{vmatrix}$ taking (1 + a^2 + b^2) common from $c_1 \text{and} c_2$ $= (1+a^2+b^2)^2 \begin{vmatrix} 1 & 0 & -2\text{b} \\ 0 & 1 & 2\text{a} \\ b & -a & 1-a^2-b^2 \end{vmatrix}$ expanding $= (1+a^2+b^2)^2[1[1-a^2-b^2+2\text{a}^2]-2\text{b}(-b)]$ $= (1+a^2+b^2)^2[1-a^2-b^2+2\text{a}^2+2\text{b}^2]$ $= (1+a^2+b^2)^2(1+a^2+b^2)$ $= (1+a^2+b^2)^3$ = RHS ans.

Solving System of Linear Equations (Matrix Method)

Q.4) Solve the equations using matrix method $x + 2y + z = 7$; $x + 3z = 11$; $2x - 3y = 1$

Sol.4) The given equation are

$x + 2y + z = 7$

$x + 0y + 3z = 11$

$2x - 3y + 0z = 1$

these equation can be written in matrix form

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 0 & 3 \\ 2 & -3 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 11 \\ 1 \end{bmatrix}$$

(or) A x = B

$\Rightarrow$ x = A^{-1} B

Where $A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 0 & 3 \\ 2 & -3 & 0 \end{bmatrix}$; $B \begin{bmatrix} 7 \\ 11 \\ 1 \end{bmatrix}$ & $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

Now

$|A| = 1(0+9) - 2(0-6) + 1(-3-0) = 9 + 12 - 3$

$|A| = 18 \neq 0$

$\therefore$ system is consistent and unique solution

Cofactors

$c_{11} = 9$; $c_{12} = -6$; $c_{14} = -3$

$c_{21} = -3$; $c_{22} = -2$; $c_{23} = 7$

$c_{31} = 6$; $c_{32} = -2$; $c_{33} = -2$

Now $Adj(A) = \begin{bmatrix} 9 & 6 & -3 \\ -3 & -2 & 7 \\ 6 & -2 & -2 \end{bmatrix}^7 = \begin{bmatrix} 9 & -3 & 6 \\ 6 & -2 & -2 \\ -3 & 7 & -2 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} . \text{AoyA}$

$A^{-1} = \frac{1}{18} \begin{bmatrix} 9 & -3 & 6 \\ 6 & -2 & -2 \\ -3 & 7 & -2 \end{bmatrix}$

We have x = A^{-1} B

$$x = \frac{1}{18} \begin{bmatrix} 9 & -3 & 6 \\ 6 & -2 & -2 \\ -3 & 7 & -2 \end{bmatrix} \begin{bmatrix} 7 \\ 11 \\ 1 \end{bmatrix}$$

$$\Rightarrow x = \frac{1}{18} \begin{bmatrix} 63 - 33 + 6 \\ 42 - 22 - 2 \\ -21 + 77 - 2 \end{bmatrix}$$

$$\Rightarrow x = \frac{1}{18} \begin{bmatrix} 36 \\ 18 \\ 54 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

$\therefore x = 2$, $y = 1$, $z = 3$ is the required solution ans.

Q.5) Solve the equations

$$\frac{2}{x} - \frac{3}{y} + \frac{3}{z} = 10 \; ; \; \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 10 \; ; \; \frac{3}{x} - \frac{1}{y} + \frac{2}{z} = 13$$

Sol.5) The given equations are

$\frac{2}{x} - \frac{3}{y} + \frac{3}{z} = 10$

$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 10$

$\frac{3}{x} - \frac{1}{y} + \frac{2}{z} = 13$

These equations can be written in matrix form

$$\begin{bmatrix} 2 & -3 & 3 \\ 1 & 1 & 1 \\ 3 & -1 & 2 \end{bmatrix} \begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} = \begin{bmatrix} 10 \\ 10 \\ 13 \end{bmatrix}$$

(or) A X = B

⇒ x = A^{-1} B

Where $A = \begin{bmatrix} 2 & -3 & 3 \\ 1 & 1 & 1 \\ 3 & -1 & 2 \end{bmatrix}$; $B = \begin{bmatrix} 10 \\ 10 \\ 13 \end{bmatrix}$; $X = \begin{bmatrix} 1/x \\ 1/y \\ 1/z \end{bmatrix}$

$|A| = 2(2+1) + 3(2-3) + 3(-1-3) = 6 - 3 - 12 = -9$

$|A| = -9 \neq 0$ ∴ system is consistent and unique solution

Cofactors

$c_{11} = 3$ $c_{12} = 1$ $c_{13} = -4$

$c_{21} = 3$ $c_{22} = -5$ $c_{23} = -7$

$c_{31} = -6$ $c_{32} = 1$ $c_{33} = 5$

∴ AdjA = $\begin{bmatrix} 3 & 3 & -6 \\ 1 & -5 & 1 \\ -4 & -7 & 5 \end{bmatrix}$

$A^2 = \frac{1}{|A|} \text{AdjA} = -\frac{1}{9} \begin{bmatrix} 3 & 3 & -6 \\ 1 & -5 & 1 \\ -4 & -7 & 5 \end{bmatrix}$

We have x A^{-1} B

X = $-\frac{1}{9} \begin{bmatrix} 3 & 3 & -6 \\ 1 & -5 & 1 \\ -4 & -7 & 5 \end{bmatrix} \begin{bmatrix} 10 \\ 10 \\ 13 \end{bmatrix}$

X = $-\frac{1}{9} \begin{bmatrix} 30 + 30 - 78 \\ 10 - 50 + 13 \\ -40 - 70 + 65 \end{bmatrix}$

$\begin{bmatrix} 1/x \\ 1/y \\ 1/z \end{bmatrix} = -\frac{1}{9} \begin{bmatrix} -18 \\ -27 \\ -45 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$

⇒ x = $\frac{1}{2}$; y = $\frac{1}{3}$ and z = $\frac{1}{3}$ is the required solution Ans……

Q.6) Find A^{-1} , where A = $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$. Hence solve the system of equations $x + 2y - 3z = -4$,

$2x + 3y + 2z = 2$ and $3x - 3y - 4z = 11$

Sol.6) We have , A = $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$

$|A| = 1(-12+6) - 2(-8-6) - 3(-6-9) = -6 + 28 + 45$

$|A| = 67 \neq 0$ ∴ (A is Invertible |consistent| unique solution)

Cofactors

$c_{11} = -6$ $c_{12} = 14$ $c_{13} = -15$

$c_{21} = 17$ $c_{22} = 5$ $c_{23} = 9$

$c_{31} = 13$ $c_{32} = -8$ $c_{33} = -1$

$\therefore\ \text{Adj(A)} = \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$

Now $\text{A}^{-1} = \frac{1}{|A|} \cdot AdjA$

$A^{-1} = \frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$ ....(1)

Given equation are

$x + y - 3z = -4$

$2x + 3y + 2z = 2$

$3x - 3y - 4z = 1$

These equation can be written in the form

$\begin{bmatrix} 1 & 1 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$

(or) A X = B ⇒ X = A^{-1} B

Where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $and\ B = \begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$

$X = \frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}\begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$ ....{A^{-1} from eq. (i)}

$X = \frac{1}{67}\begin{bmatrix} 24 + 34 + 143 \\ -56 + 10 - 88 \\ 60 + 18 - 11 \end{bmatrix} = \frac{1}{67}\begin{bmatrix} 201 \\ -134 \\ 67 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$

∴ x = 3 , y = –2 , z = 1 is the required solution ans.

Q.7) If = $\begin{bmatrix} 1 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$. Find A–1 and hence solve the equation $x + 2y + z = 4$; $-x + y + z = 0$ and $x - 3y + z = 2$.

Sol.7) We have A = $\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$

Do yourself

$|A| = 10 \neq 0$ ∴ (A is invertible)

$AdjA = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$

$\text{A}^{-1} = \frac{1}{|\text{A}|} \cdot AdjA$

$\text{A}^{-1} = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$ .....(1)

Given equation are

$x + 2y + z = 4$

$-x + y + z = 0$

$x - 3y + z = 2$

→ the matrix of above equation is clearly the transpose of given matrix A

∴ these equations can be written in the form

A' X = B where B = $\begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$; $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

⇒ X = $(A^{-1})^{-1}$B

$\Rightarrow$ X = $(A^{-1})^{-1}B$ ....{By prop. $(A^{-1})^{-1} = (A^{-1})^{1}$}

$$\Rightarrow X = \frac{1}{10}\begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

$$\Rightarrow X = \frac{1}{10}\begin{bmatrix} 16+2 \\ 8-4 \\ 8+6 \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 18 \\ 4 \\ 14 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9/5 \\ 2/5 \\ 7/5 \end{bmatrix}$$

$\Rightarrow x = \frac{9}{5}$, $y = \frac{2}{5}$, $z = \frac{7}{5}$ is the req. solution ans..

Q.8) Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and hence (or) use it to solve the equations $x - y + z = 4$; $x - 2y - 2z = 9$; $2x + y + 3z = 1$

Sol.8) Let $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$

$$CA = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$$

$\Rightarrow CA = 8\,I$

Post multiply by A^{-1}

$\Rightarrow CAA^{-1} = 8\,I\,A^{-1}$

$\Rightarrow CI = 8A^{-1}$ ....$\left\{\begin{matrix} AA^{-1} = I \\ IA^{-1} = A^{-1} \end{matrix}\right\}$

$$\Rightarrow A^{-1} = \frac{1}{8}C = \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$$

Given equation are

$x - y + z = 4$

$x - 2y - 2z = 9$

$2x + y + 3z = 1$

There equation can be in the form

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & 2 \\ 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

(or) $A\,X = B$

$\Rightarrow X = A^{-1}B$ where B = $\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$; X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$$\Rightarrow X = \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

$$\Rightarrow X = \frac{1}{8}\begin{bmatrix} 24 \\ -16 \\ -8 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$$

$\Rightarrow x = 3 , y = -2$ and $z = -1$ is the req. solution ans.

Q.9) A = $\begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}$ find AB and hence solve the equations

$x - 2y = 0$; $2x + y + 3z = 8$; $-2y + x = 7$

Sol.9) $$AB = \begin{bmatrix}1 & -2 & 0\\2 & 1 & 3\\0 & -2 & 1\end{bmatrix}\begin{bmatrix}7 & 2 & -6\\-2 & 1 & -3\\-4 & 2 & 5\end{bmatrix} = \begin{bmatrix}11 & 0 & 0\\0 & 11 & 0\\0 & 0 & 11\end{bmatrix}$$

$\Rightarrow AB = 11\,I$

Pre by A^{-1}

$\Rightarrow A^{-1}AB = 11\,A^{-1}I$

$\Rightarrow I\,B = 11\,A^{-1}$

$\Rightarrow B = 11\,A^{-1}$

$$\Rightarrow A^{-1} = \frac{1}{11}B1 = \frac{1}{11}\begin{bmatrix}7 & 2 & -6\\-2 & 1 & -3\\-4 & 2 & 5\end{bmatrix}$$

Given equations are

x – 2y = 10

2x + y + 3z = 7

–2y + 0y + z = 7

These equations can be written in the form

$A\,X = C \quad \Rightarrow \quad X = A^{-1}\,C$

Where $X = \begin{bmatrix}x\\y\\z\end{bmatrix}$ and $C = \begin{bmatrix}10\\8\\7\end{bmatrix}$

$$X = \frac{1}{11}\begin{bmatrix}7 & 2 & -6\\-2 & 1 & -3\\-4 & 2 & 5\end{bmatrix}\begin{bmatrix}10\\8\\7\end{bmatrix}$$

$$\Rightarrow X = \frac{1}{11}\begin{bmatrix}70+16-42\\-20+8-21\\-40+16+35\end{bmatrix} = \frac{1}{11}\begin{bmatrix}44\\-33\\11\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}4\\-3\\1\end{bmatrix}$$

$\therefore\ x = 4\ ,\ y = -3\ ,\ z = 1$ is the required solution ans.

Q.10) Show that system of equations is consistent and also find the solution

$2x - y + 3z = 5\ ;\ 3x + 2y - z = 7\ ;\ 4x + 5y - 5z = 9$

Sol.10) Given equation are

$2x - y + 3z = 5$

$3x + 2y - z = 7$

$4x + 5y - 5z = 9$

given equation can be written in the form

$A\,X = B \quad \Rightarrow \quad X = A^{-1}B$

where $A = \begin{bmatrix}2 & -1 & 3\\3 & 2 & -1\\4 & 5 & -5\end{bmatrix}$; $X = \begin{bmatrix}x\\y\\z\end{bmatrix}$ & $B = \begin{bmatrix}5\\7\\9\end{bmatrix}$

|A| = 0 {solution can be infinite many or no solution}

Now $AdjA = \begin{bmatrix}-5 & 10 & -5\\11 & -22 & 11\\7 & -14 & 7\end{bmatrix}$

Now $(AdjA)B = \begin{bmatrix}-5 & 10 & -5\\11 & -12 & 11\\7 & -14 & 7\end{bmatrix}\begin{bmatrix}5\\7\\9\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix} = 0$

Since |A| = 0 also (AdjA). B = 0

∴ System is consistent and Infinite many solutions

→ Put $z = k$ in first two equations, we get

$2x - y = 5 - 3k \qquad \ldots..(k \in R)$

$3x + 2y = 7 + k$

(or) $\begin{bmatrix}2 & -1\\3 & 2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}5-3k\\7+k\end{bmatrix}$

$$A = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix} \;;\; X = \begin{bmatrix} x \\ y \end{bmatrix} \;;\; B = \begin{bmatrix} 5-3k \\ 7+k \end{bmatrix}$$

$|A| = 7$ and $AdjA = \begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$

$$A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$$

$$X = \mathrm{A}^{-1}B$$

$$X = \frac{1}{7}\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 5-3k \\ 7+k \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 17-5k \\ 11k-1 \end{bmatrix}$$

$\therefore\; x = \frac{17-5k}{7} \;;\; y = \frac{11k-1}{7}$ and $z = k$ ans.