

Determinants Class 12th

Q.1)	Show $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix} = 0$
Sol.1)	$\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$ $C_3 \rightarrow C_3 + C_2$ $= \begin{vmatrix} 1 & a & a+b+c \\ 1 & b & a+b+c \\ 1 & c & a+b+c \end{vmatrix}$ <p style="margin-left: 20px;">taking (a + b + c) common from C₃</p> $= (a+b+c) \begin{vmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{vmatrix}$ $= (a+b+c) \times 0 = 0 \quad \dots\{\because C_1 \& C_3 \text{ are identical}\}$
Q.2)	Show that $\begin{vmatrix} b-c & c-a & a-b \\ c-a & a-b & b-c \\ a-b & b-c & c-a \end{vmatrix} = 0$
Sol.2)	$\text{let } \Delta = \begin{vmatrix} b-c & c-a & a-b \\ c-a & a-b & b-c \\ a-b & b-c & c-a \end{vmatrix}$ $c_1 \rightarrow c_1 + c_2 + c_3$ $= \begin{vmatrix} 0 & c-a & a-b \\ 0 & a-b & b-c \\ 0 & b-c & c-a \end{vmatrix}$ $= 0 \quad \dots\{\text{all elements of } C_1 \text{ are } 0\}$
Q.3)	Show that $\begin{vmatrix} \sin\alpha & \cos\alpha & \sin(\alpha+S) \\ \sin\beta & \cos\beta & \sin(\beta+S) \\ \sin\gamma & \cos\gamma & \sin(\gamma+S) \end{vmatrix} = 0$
Sol.3)	$\Delta = \begin{vmatrix} \sin\alpha & \cos\alpha & \sin\alpha \cdot \cos S + \cos\alpha \cdot \sin S \\ \sin\beta & \cos\beta & \sin\beta \cdot \cos S + \cos\beta \cdot \sin S \\ \sin\gamma & \cos\gamma & \sin\gamma \cdot \cos S + \cos\gamma \cdot \sin S \end{vmatrix}$ <p style="margin-left: 20px;">Applying sum property in C₃</p> $= \begin{vmatrix} \sin\alpha & \cos\alpha & \sin\alpha \cdot \cos S \\ \sin\beta & \cos\beta & \sin\beta \cdot \cos S \\ \sin\gamma & \cos\gamma & \sin\gamma \cdot \cos S \end{vmatrix} + \begin{vmatrix} \sin\alpha & \cos\alpha & \cos\alpha \cdot \sin S \\ \sin\beta & \cos\beta & \cos\beta \cdot \sin S \\ \sin\gamma & \cos\gamma & \cos\gamma \cdot \sin S \end{vmatrix}$ $= \cos S \begin{vmatrix} \sin\alpha & \cos\alpha & \sin\alpha \\ \sin\beta & \cos\beta & \sin\beta \\ \sin\gamma & \cos\gamma & \sin\gamma \end{vmatrix} + \sin S \begin{vmatrix} \sin\alpha & \cos\alpha & \cos\alpha \\ \sin\beta & \cos\beta & \cos\beta \\ \sin\gamma & \cos\gamma & \cos\gamma \end{vmatrix}$ $= \cos S(0) + \sin S(0) = 0 \quad \text{ans.}$

Q.4)	If a, b, c are in A.P find value of $\begin{vmatrix} 2y + 45y + 78y + a \\ 3y + 56y + 89y + b \\ 4y + 67y + 910y + c \end{vmatrix}$
Sol.4)	<p>let $\Delta = \begin{vmatrix} 2y + 45y + 78y + a \\ 3y + 56y + 89y + b \\ 4y + 67y + 910y + c \end{vmatrix}$</p> <p>given a, b, c are in A.P $a + c = 2b$</p> <p>$\therefore R_1 \rightarrow R_1 + R_3$</p> $= \begin{vmatrix} 6y + 1012y + 1618y + a + c \\ 3y + 56y + 89y + b \\ 4y + 67y + 910y + c \end{vmatrix}$ $= \begin{vmatrix} 6y + 1012y + 1618y + 2b \\ 3y + 56y + 89y + b \\ 4y + 67y + 910y + c \end{vmatrix} \quad \dots\{\because a + c = 2b\}$ <p>taking 2 common from R_1</p> $= 2 \begin{vmatrix} 3y + 56y + 89y + b \\ 3y + 56y + 89y + b \\ 4y + 67y + 910y + c \end{vmatrix}$ <p>clearly R_1 and R_2 are identical</p> <p>$\therefore 2 \times 0 = 0$ ans.</p>
Q.5)	Show that $\begin{vmatrix} (a^x + a^{-x})^2(a^x - a^{-x})^2 1 \\ (a^y + a^{-y})^2(a^y - a^{-y})^2 1 \\ (a^z + a^{-z})^2(a^z - a^{-z})^2 1 \end{vmatrix} = 0$
Sol.5)	<p>$C_1 \rightarrow C_1 - C_2$</p> $= \begin{vmatrix} (a^x + a^{-x})^2 - (a^x - a^{-x})^2(a^x - a^{-x})^2 1 \\ (a^y + a^{-y})^2 - (a^y - a^{-y})^2(a^y - a^{-y})^2 1 \\ (a^z + a^{-z})^2 - (a^z - a^{-z})^2(a^z - a^{-z})^2 1 \end{vmatrix}$ $= \begin{vmatrix} 4(a^x - a^{-x})^2 1 \\ 4(a^y - a^{-y})^2 1 \\ 4(a^z - a^{-z})^2 1 \end{vmatrix} \quad \dots\{(a + b)^2 - (a - b)^2 = 4ab\}$ <p>4}</p> $= 4 \begin{vmatrix} 1(a^x - a^{-x})^2 1 \\ 1(a^y - a^{-y})^2 1 \\ 1(a^z - a^{-z})^2 1 \end{vmatrix}$ $= 4 \times 0 = 0 \quad \dots\{C_1 \& C_3 \text{ are identical}\}$
Q.6)	Show that $\begin{vmatrix} 4115 \\ 7979 \\ 2953 \end{vmatrix} = 0$
Sol.6)	let $\Delta = \begin{vmatrix} 4115 \\ 7979 \\ 2953 \end{vmatrix}$

	$c_2 \rightarrow c_2 + 8c_3$ $= \begin{vmatrix} 41415 \\ 79799 \\ 29293 \end{vmatrix} = 0 \quad \dots \{C_1 \& C_3 \text{ are identical}\}$
Q.7)	Show $\begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & a+b \\ a^2b^2 & ab & a+b \end{vmatrix} = 0$
Sol.7)	$R_1 \rightarrow aR_1, R_2 \rightarrow bR_2 \text{ and } R_3 \rightarrow cR_3$ $= \frac{1}{abc} \begin{vmatrix} ab^2c^2 & abc & ab+ac \\ bc^2a^2 & abc & bc+ab \\ ca^2b^2 & abc & ca+bc \end{vmatrix}$ <p>taking abc common from C_1 and C_2</p> $= \frac{1}{abc} \cdot (abc)(abc) \begin{vmatrix} bc & 1 & ab+ac \\ ca & 1 & bc+ab \\ ab & 1 & ca+bc \end{vmatrix}$ $c_3 \rightarrow c_3 + c_1$ $= abc \begin{vmatrix} bc & 1 & ab+bc+ca \\ ca & 1 & ab+bc+ca \\ ab & 1 & ab+bc+ca \end{vmatrix}$ <p>taking $(ab+bc+ca)$ common from C_3</p> $= abc(ab+bc+ca) \begin{vmatrix} bc & 1 & 1 \\ ca & 1 & 1 \\ ab & 1 & 1 \end{vmatrix}$ $= abc(ab+bc+ca)(0) = 0 \quad \dots \dots \{C_1 \& C_3 \text{ are identical}\}$
Q.8)	Show that $\begin{vmatrix} 1 & a & a^2 - bc \\ 1 & b & b^2 - ca \\ 1 & c & c^2 - ab \end{vmatrix} = 0$
Sol.8)	$\text{let } \Delta = \begin{vmatrix} 1 & a & a^2 - bc \\ 1 & b & b^2 - ca \\ 1 & c & c^2 - ab \end{vmatrix}$ <p>Applying sum property in C_3</p> $= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$ $R_1 \rightarrow aR_1, R_2 \rightarrow bR_2, R_3 \rightarrow cR_3$ $= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^3 \end{vmatrix} - \frac{1}{abc} \begin{vmatrix} a & a^2 & abc \\ b & b^2 & abc \\ c & c^2 & abc \end{vmatrix}$ <p>taking abc common from C_3 in 2nd Det.</p> $= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \frac{abc}{abc} \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix}$ $c_2 \leftrightarrow c_3$

	$= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} + \begin{vmatrix} a & 1 & a^2 \\ b & 1 & b^2 \\ c & 1 & b^2 \end{vmatrix}$ <p>again $c_1 \leftrightarrow c_2$</p> $= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ $= 0 \quad \text{ans.}$
Q.9)	If a , b , c are the P^{th} , q^{th} and r^{th} terms of G.P then show that $\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix} = 0$
Sol.9)	<p>We know n^{th} term of G.P : $a_n = ar^{n-1}$</p> <p>here let $A \rightarrow I^{st}$ term and R common ratio</p> <p>$\therefore a_p = a = AR^{p-1}$; $a_q = b = AR^{q-1}$; $a_r = c = AR^{r-1}$</p> <p>taking log on both sides</p> <p>$\log a = \log(AR^{p-1})$; $\log b = \log(AR^{q-1})$; $\log c = \log(AR^{r-1})$</p> <p>$\Rightarrow \log a = \log A + (p-1)\log R$</p> <p>$\log b = \log A + (q-1)\log R$</p> <p>$\log c = \log A + (r-1)\log R$</p> <p>Now let $\Delta = \begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix}$</p> <p>putting values of $\log a$, $\log b$ and $\log c$</p> $= \begin{vmatrix} \log A + (p-1)\log R & p & 1 \\ \log A + (q-1)\log R & q & 1 \\ \log A + (r-1)\log R & r & 1 \end{vmatrix}$ <p>Applying sum property in c_1</p> $= \begin{vmatrix} \log A & p & 1 \\ \log A & q & 1 \\ \log A & r & 1 \end{vmatrix} + \begin{vmatrix} (p-1)\log R & p & 1 \\ (q-1)\log R & q & 1 \\ (r-1)\log R & r & 1 \end{vmatrix}$ $= \log A \begin{vmatrix} 1 & p & 1 \\ 1 & q & 1 \\ 1 & r & 1 \end{vmatrix} + \log R \begin{vmatrix} p-1 & p & 1 \\ q-1 & q & 1 \\ r-1 & r & 1 \end{vmatrix}$ <p>$c_1 \rightarrow c_1 + c_3$</p> $= \log A \begin{vmatrix} 1 & p & 1 \\ 1 & q & 1 \\ 1 & r & 1 \end{vmatrix} + \log R \begin{vmatrix} p & p & 1 \\ q & q & 1 \\ r & r & 1 \end{vmatrix}$ $= \log A \times (0) + \log R \times (0)$ $= 0 + 0 = 0 \quad \text{ans.}$
Q.10)	Show $\begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix} = \begin{vmatrix} y & b & q \\ x & a & p \\ z & c & r \end{vmatrix}$

Sol.10)

$$\begin{aligned} \text{let } \Delta &= \begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix} \\ &= \begin{vmatrix} a & x & p \\ b & y & q \\ c & z & r \end{vmatrix} \quad \dots \{ \cdot, |A| = |A'| \} \end{aligned}$$

$$c_1 \leftrightarrow c_2$$

$$= - \begin{vmatrix} x & a & p \\ y & b & q \\ z & c & r \end{vmatrix}$$

$$R_2 \leftrightarrow R_1$$

$$= (-)(-) \begin{vmatrix} y & b & q \\ x & a & p \\ z & c & r \end{vmatrix}$$

$$= \begin{vmatrix} y & b & q \\ x & a & p \\ z & c & r \end{vmatrix} = \text{RHS} \quad \text{Proved}$$