

CLASS XII
Continuity and Differentiability

One marks questions

Differentiate the following with respect to x:

1. $2^{\cos^2 x}$ Ans:- $2^{\cos^2 x} \sin 2x \log 2$ (NCERT EXEMPLAR)
2. $\log(x + \sqrt{a + x^2})$ Ans: $\frac{1}{\sqrt{x^2+a}}$ (NCERT EXEMPLAR)
3. $\sqrt{e^{\sqrt{x}}}$ Ans: $\frac{e^{\sqrt{x}}}{4\sqrt{x}e^{\sqrt{x}}}$

Four marks questions

1. If $\sin y = x \sin(a + y)$, prove that $\frac{dy}{dx} = \frac{\sin^2(a + y)}{\sin a}$
2. If $y = x \sin y$, prove that $\frac{dy}{dx} = \frac{y}{x(1 - x \cos y)}$.
3. If $\sqrt{1 - x^6} + \sqrt{1 - y^6} = a(x^3 - y^3)$, prove that $\frac{dy}{dx} = \frac{x^2}{y^2} \sqrt{\frac{1 - y^6}{1 - x^6}}$
4. If $x = a(\cos t + t \sin t)$ and $y = a(\sin t - t \cos t)$, find the value of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$, when $t = \frac{\pi}{4}$.
5. If $y = \log[x + \sqrt{x^2 + 1}]$, prove that $(x^2 + 1)y_2 + xy_1 = 0$.
6. If $y = e^{a \cos^{-1} x}$, $-1 \leq x \leq 1$, show that $(1 - x^2)y_2 - xy_1 - a^2 y = 0$.
7. Differentiate with respect to x:
 a) $\tan^{-1}\left(\frac{\sqrt{1 + \sin x} + \sqrt{1 - \sin x}}{\sqrt{1 + \sin x} - \sqrt{1 - \sin x}}\right)$ Ans: $\frac{dy}{dx} = -\frac{1}{2}$
8. If $y = \sin^{-1}\left(\frac{2x}{1 + x^2}\right) + \sec^{-1}\left(\frac{1 + x^2}{1 - x^2}\right)$, prove that $\frac{dy}{dx} = \frac{4}{1 + x^2}$.
9. Verify Rolle's theorem for the function $f(x) = \sin x - \sin 2x$ on $[0, \pi]$.
10. Verify Mean Value theorem for the function $f(x) = x^2 - 2x + 4$ on $[1, 5]$.
11. Find the values of 'a' and 'b' when

$$f(x) = \begin{cases} 3ax + b, & x > 1 \\ 11, & x = 1 \\ 5ax - 2b, & x < 1 \end{cases}$$
 is continuous at $x = 1$. Ans: $a = 3, b = 2$.
12. If $x^{13} \cdot y^7 = (x + y)^{20}$, prove that $\frac{dy}{dx} = \frac{y}{x}$.
13. If $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \dots \text{to } \infty}}}$, prove that $\frac{dy}{dx} = \frac{\cos x}{2y - 1}$.

14. Check, the function $f(x) = \begin{cases} |x - a| \sin \frac{1}{x - a}, & \text{if } x \neq 0 \\ 0, & \text{if } x = a \end{cases}$ is continuous or discontinuous at $x = a$. (NCERT EXEMPLAR).

15. Examine the differentiability of f , where f is defined by

$$f(x) = \begin{cases} x[x], & \text{if } 0 \leq x < 2 \\ (x - 1)x, & \text{if } 2 \leq x < 3 \end{cases} \quad \text{at } x = 2, \text{ (NCERT EXEMPLAR).}$$

16. Find $\frac{dy}{dx}$ of the function expressed in parametric form give $x = \frac{1 + \log t}{t^2}, y = \frac{3 + 2 \log t}{t}$.

(NCERT EXEMPLAR). Ans : t

17. Find $\frac{dy}{dx}$ when x and y are connected by the relation given by $\sin(xy) + \frac{x}{y} = x^2 - y$

(NCERT EXEMPLAR). Ans: $\frac{2xy^2 - y^3 \cos(xy) - y}{xy^2 \cos(xy) - x + y^2}$

18. Discuss the applicability of Rolle's Theorem on the function given

$$\text{by } f(x) = \begin{cases} x^2 + 1, & \text{if } 0 \leq x \leq 1 \\ 3 - x, & \text{if } 1 \leq x \leq 2 \end{cases}.$$

(NCERT EXEMPLAR)