

DEFINITE INTEGRALS

Q.1)	$I = \int_{-1}^2 x^3 - x dx$
Sol.1)	$I = \int_{-1}^2 \frac{ x(x+1)(x-1) }{(0)(-1)(1)} dx$ $\therefore I = + \int_{-1}^0 (x^3 - x) dx - \int_0^1 (x^3 - x) dx + \int_1^2 (x^3 - x) dx$ $I = \left[\frac{x^4}{4} - \frac{x^2}{2} \right]_{-1}^0 - \left[\frac{x^4}{4} - \frac{x^2}{2} \right]_0^1 + \left[\frac{x^4}{4} - \frac{x^2}{2} \right]_1^2$ $I = \left[(0) - \left(\frac{1}{4} - \frac{1}{2} \right) \right] - \left[\left(\frac{1}{4} - \frac{1}{2} \right) - (0) \right] + \left[(4 - 2) - \left(\frac{1}{4} - \frac{1}{2} \right) \right]$ $I = \frac{11}{4} \quad \text{ans.}$
Q.2)	$I = \int_1^4 \frac{ x-3 }{(3)} + \frac{ x-1 }{(1)} + \frac{ x-2 }{(2)} dx$
Sol.2)	$I = \int_1^4 \frac{ x-1 }{(1)} + \frac{ x-2 }{(2)} + \frac{ x-3 }{(3)} dx$ $I = \int_1^2 (x-1) - (x-2) - (x-3) dx + \int_2^3 (x-1) + (x-2) - (x-3) dx +$ $\int_3^4 (x-1) + (x-2) + (x-3) dx$ $I = \int_1^2 (-x+4) dx + \int_2^3 x dx + \int_3^4 (3x-6) dx$ $I = \left[\frac{-x^2}{2} + 4x \right]_1^2 + \left[\frac{x^2}{2} \right]_2^3 + \left[\frac{3x^2}{2} - 6x \right]_3^4$ $I = \left[(-2+8) - \left(\frac{-1}{2} + 4 \right) \right] + \left[\frac{9}{2} - 2 \right] + \left[(24-24) - \left(\frac{27}{2} - 18 \right) \right]$ $I = 6 - \frac{7}{2} + \frac{5}{2} + \frac{9}{2}$ $I = \frac{19}{2} \quad \text{ans.}$
Q.3)	$I = \int_0^{2\pi} \sin x dx$
Sol.3)	$I = \int_0^{\pi} \sin x dx - \int_{\pi}^{2\pi} \sin x dx$ $I = [-\cos x]_0^{\pi} - [-\cos x]_{\pi}^{2\pi}$ $I = -[\cos x]_0^{\pi} + [\cos x]_{\pi}^{2\pi}$ $I = -[\cos \pi - \cos 0] + [\cos 2\pi - \cos \pi]$ $I = -[-1 - 1] + [0 - (-1)]$ $I = 2 + 2$ $I = 4 \quad \text{ans.}$
Q.4)	$I = \int_0^{\frac{\pi}{2}} \cos(2x) dx$
Sol.4)	$I = \int_0^{\frac{\pi}{2}} \cos(2x) dx$

	<p>Critical point $2x = \frac{\pi}{2} \quad x = \frac{\pi}{4}$</p> <p>$\therefore I = \int_0^{\frac{\pi}{4}} \cos(2x) dx - \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos(2x) dx$</p> <p>$\left\{ \begin{array}{l} \because 0 < x < \pi/4 \quad \because \pi/4 < x < \pi/2 \\ 0 < 2x < \pi/2 \quad \pi/2 < 2x < \pi \end{array} \right\}$</p> <p>1st quad (+ve) 2nd quad (-ve)</p> <p>$I = \frac{1}{2} [\sin(2x)]_0^{\pi/4} - \frac{1}{2} [\sin(2x)]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$</p> <p>$I = \frac{1}{2} \left[\left(\sin \frac{\pi}{2} - \sin 0 \right) \right] - \frac{1}{2} \left[\sin \pi - \sin \frac{\pi}{2} \right]$</p> <p>$I = \frac{1}{2} [1 - 0] - \frac{1}{2} [0 - 1]$</p> <p>$I = \frac{1}{2} + \frac{1}{2}$</p> <p>$I = 1 \quad \text{ans.}$</p>
Q.5)	$I = \int_0^3 [x] dx$ (greatest integer function)
Sol.5)	<p>$I = \int_0^1 [x] dx + \int_1^2 [x] dx + \int_2^3 [x] dx$</p> <p>$I = \int_0^1 (0) dx + \int_1^2 (1) dx + \int_2^3 (2) dx$</p> <p>$I = 0 + (x)_1^2 + (2x)_2^3$</p> <p>$I = (2 - 1) + (6 - 4)$</p> <p>$I = 3 \quad \text{ans.}$</p>
Q.6)	$I = \int_0^2 [x^2] dx$
Sol.6)	<p>$I = \int_0^2 [x^2] dx$</p> <p>$I = \int_0^1 [x^2] dx + \int_1^{\sqrt{2}} [x^2] dx + \int_{\sqrt{2}}^{\sqrt{3}} [x^2] dx + \int_{\sqrt{3}}^2 [x^2] dx$</p> <p>$I = \int_0^1 (0) dx + \int_1^{\sqrt{2}} (1) dx + \int_{\sqrt{2}}^{\sqrt{3}} (2) dx + \int_{\sqrt{3}}^2 (3) dx$</p> <p>$I = 0 + (x)_1^{\sqrt{2}} + (2x)_{\sqrt{2}}^{\sqrt{3}} + (3x)_{\sqrt{3}}^2$</p> <p>$I = (\sqrt{2} - 1) + (2\sqrt{3} - 2\sqrt{2}) + (6 - 3\sqrt{3})$</p> <p>$I = 5 - \sqrt{2} - \sqrt{3} \quad \text{Ans ...}$</p>
Q.7)	$I = \int_0^{1.5} [x^2] dx$
Sol.7)	<p>$I = \int_0^{1.5} [x^2] dx$</p> <p>$I = \int_0^1 [x^2] dx + \int_1^{\sqrt{2}} [x^2] dx + \int_{\sqrt{2}}^{1.5} [x^2] dx$</p> <p>$I = \int_0^1 0 dx + \int_1^{\sqrt{2}} (1) dx + \int_{\sqrt{2}}^{1.5} (2) dx$</p> <p>$I = 0 + (x)_1^{\sqrt{2}} + (2x)_{\sqrt{2}}^{1.5}$</p>

	$I = (\sqrt{2} - 1) + (3 - 2\sqrt{2})$ $I = 2 - \sqrt{2} \quad \text{ans.}$
Q.8)	$I = \int_{-1}^{\frac{3}{2}} x \sin(\pi x) dx$
Sol.8)	<p>Case - I</p> $-1 < x < 0 \quad ; \quad -\pi < \pi x < 0$ $ x \sin(\pi x) = x \sin(\pi x)$ <p>Case-II</p> $1 < x < 1 \quad ; \quad 0 < \pi x < \pi$ $ x \sin(\pi x) = x \sin(\pi x)$ <p>Case-III</p> $1 < x < \frac{3}{2} \quad ; \quad \pi < \pi x < \frac{3\pi}{2}$ $ x \sin(\pi x) = -x \sin(\pi x)$ $\therefore I = \int_{-1}^1 x \sin(\pi x) dx - \int_1^{\frac{3}{2}} x \sin(\pi x) dx$ <p>let $I_1 = \int x \sin(\pi x) dx$</p> $I_1 = \left[\frac{-x \cos(\pi x)}{\pi} - \int (1) \frac{(-\cos \pi x)}{\pi} dx \right]$ $I_1 = \frac{-x \cos(\pi x)}{\pi} + \frac{1}{\pi} \int \cos(\pi x) dx$ $I_1 = \frac{-x \cos(\pi x)}{\pi} + \frac{\sin(\pi x)}{\pi^2}$ $\therefore I = \left[\frac{-x \cos(\pi x)}{\pi} + \frac{\sin(\pi x)}{\pi^2} \right]_{-1}^1 - \left[\frac{-x \cos(\pi x)}{\pi} + \frac{\sin(\pi x)}{\pi^2} \right]_1^{\frac{3}{2}}$ $I = \left[\left(\frac{-\cos(\pi)}{\pi} + \frac{\sin(\pi)}{\pi^2} \right) - \left(\frac{\cos(-\pi)}{\pi} + \frac{\sin(-\pi)}{\pi^2} \right) \right] - \left[\left(\frac{-3}{2} \frac{\cos(\frac{3\pi}{2})}{\pi} + \frac{\sin(\frac{3\pi}{2})}{\pi^2} \right) - \left(\frac{-\cos(\pi)}{\pi} + \frac{\sin(\pi)}{\pi^2} \right) \right]$ $I = \left[\left(\frac{1}{\pi} + 0 \right) - \left(\frac{-1}{\pi} + 0 \right) \right] - \left[\left(0 - \frac{1}{\pi^2} \right) - \left(\frac{+1}{\pi} + 0 \right) \right] \quad \dots \begin{cases} \cos(\pi) = -1 & ; & \cos(-\pi) = -1 \\ \sin(-\pi) = 0 & ; & \sin(\frac{3\pi}{2}) = -1 \\ \cos(3\pi/2) = 0 & & \end{cases}$ $I = \frac{1}{\pi} + \frac{1}{\pi} + \frac{1}{\pi^2} + \frac{1}{\pi}$ $\therefore I = \frac{3}{\pi} + \frac{1}{\pi^2} \quad \text{ans.}$
Q.9)	$I = \int_{1/e}^e \log x dx$
Sol.9)	$I = \int_{1/e}^e \log x dx$ <p>Critical $x = 1$ since $\log 1 = 0$</p> $\therefore I = -\int_{1/e}^1 \log x dx + \int_1^e \log x dx$ <p>let $I_1 = \int \log x dx$</p> $= \int \log x \cdot 1 dx$

	$= \int \log x \cdot x - \int \frac{1}{x} \cdot x dx$ $I_1 = x \log x - x$ $\therefore I = -[x \log x - x]_{1/e}^1 + [x \log x - x]_1^e$ $= -\left[(\log 1 - 1) - \left(\frac{1}{e} \log \left(\frac{1}{e}\right) - \frac{1}{e}\right)\right] + [(e \log e - e) - (\log 1 - 1)]$ $= -\left[(0 - 1) - \left(\frac{-1}{e} - \frac{1}{e}\right)\right] + [(e - e) - (0 - 1)] \quad \dots \dots \left\{ \because \log = 1, \log \left(\frac{1}{e}\right) = -1 \right\}$ $= 1 - \frac{2}{e} + 1$ $= 2 - \frac{2}{e} \quad \text{ans.}$
Q.10)	$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x dx$
Sol.10)	$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x dx$ <p>Here</p> $f(x) = \sin^7 x$ $f(-x) = \sin^7(-x) = (-\sin x)^7 = -f(x)$ $\therefore f(-x) = -f(x)$ <p>$f(x) \rightarrow$ odd function</p> $\therefore I = 0 \quad \text{Ans.} \quad \dots \dots \left\{ \int_{-a}^a f(x) dx = 0 \text{ when } f(x) \text{ is odd function} \right\}$