

DEFINITE INTEGRALS

Q.1)	$I = \int_0^{\pi} \log (1 + \cos x) dx$
Sol.1)	$I = \int_0^{\pi} \log (1 + \cos x) dx \quad \dots(1)$ $I = \int_0^{\pi} \log [1 + \cos (\pi - x)] dx \quad \dots(P-IV)$ $I = \int_0^{\pi} \log (1 - \cos x) dx \quad \dots(2)$ <p>(1) + (2)</p> $2I = \int_0^{\pi} \log ((1 + \cos x)(1 - \cos x)) dx$ $2I = \int_0^{\pi} \log (1 - \cos^2 x) dx$ $2I = \int_0^{\pi} \log (\sin^2 x) dx$ $2I = 2 \int_0^{\pi} \log (\sin x) dx \quad \dots[\log m^n = n \log m]$ $I = \int_0^{\pi} \log (\sin x) dx$ $I = 2 \int_0^{\frac{\pi}{2}} \log (\sin x) dx \quad \dots(P-VI)$ $\frac{1}{2} = \int_0^{\frac{\pi}{2}} \log (\sin x) dx \quad \dots(3)$ $\frac{1}{2} = \int_0^{\frac{\pi}{2}} \log \left(\sin \left(\frac{\pi}{2} - x \right) \right) dx \quad \dots(P-IV)$ $\frac{1}{2} = \int_0^{\frac{\pi}{2}} \log (\cos x) dx \quad \dots(4)$ <p>(3) + (4)</p> $I = \int_0^{\frac{\pi}{2}} \log (\sin x \cdot \cos x) dx$ $I = \int_0^{\frac{\pi}{2}} \log \left(\frac{\sin(2x)}{2} \right) dx$ $I = \int_0^{\frac{\pi}{2}} \log (\sin(2x)) - \log 2 dx$ $I = \int_0^{\frac{\pi}{2}} \log (\sin(2x)) dx - \int_0^{\frac{\pi}{2}} \log 2 dx$ $I = \int_0^{\frac{\pi}{2}} \log (\sin(2x)) dx - \log_2(x) \Big _0^{\frac{\pi}{2}}$ $I = \int_0^{\frac{\pi}{2}} \log (\sin(2x)) dx - \frac{\pi}{2} \log 2$ $I = I_1 - \frac{\pi}{2} \log 2 \quad \dots(5)$ <p>Where $I_1 = \int_0^{\frac{\pi}{2}} \log (\sin(2x)) dx$</p> <p>Put $2x = t$ when $x = 0$; $t = 0$</p> $dx = \frac{dt}{2} \quad \quad \quad x = \frac{\pi}{2} ; t = \pi$ <p>$\therefore I_1 = \frac{1}{2} \int_0^{\pi} \log (\sin t) dx$</p>

	$I_1 = \frac{1}{2} \times 2 \int_0^{\frac{\pi}{2}} \log(\sin t) dt \quad \dots(\text{P-VI})$ $I_1 = \int_0^{\frac{\pi}{2}} \log(\sin t) dt$ $I_1 = \int_0^{\frac{\pi}{2}} \log(\sin x) dx \quad \dots\dots(\text{P-I})$ $I_1 = \frac{1}{2} \quad \dots\dots\{\text{from eq. 3}\}$ <p>\therefore eq. (5) becomes</p> $I = \frac{1}{2} - \frac{\pi}{2} \log 2$ $\Rightarrow I = \frac{1}{2} = \frac{-\pi}{2} \log 2$ $\Rightarrow \frac{1}{2} = \frac{-\pi}{2} \log 2$ $I = -\pi \log 2 \quad \text{ans.}$
Q.2)	$I = \int_0^{\pi} \frac{x \tan x}{\sec x \cdot \operatorname{cosec} x} dx$
Sol.2)	$I = \int_0^{\pi} \frac{\frac{x \sin x}{\cos x}}{\frac{1}{\cos x} \cdot \frac{1}{\sin x}} dx$ $I = \int_0^{\pi} x \sin^2 x dx \quad \dots\dots(1)$ $I = \int_0^{\pi} (\pi - x) \sin^2(\pi - x) dx \quad \dots\dots(\text{P-IV})$ $I = \int_0^{\pi} (\pi - x) \sin^2 x dx \quad \dots\dots(2)$ <p>(1) + (2)</p> $2I = \int_0^{\pi} x \sin^2 x + \pi \sin^2 x - x \sin^2 x dx$ $2I = \pi \int_0^{\pi} \sin^2 x dx$ $2I = 2\pi \int_0^{\frac{\pi}{2}} \sin^2 x dx \quad \dots\dots(\text{P-VI})$ $I = \pi \int_0^{\frac{\pi}{2}} \sin^2 x dx$ $I = \pi \int_0^{\frac{\pi}{2}} \frac{1 - \cos(2x)}{2} dx$ $I = \frac{\pi}{2} \left[x - \frac{\sin(2x)}{2} \right]_0^{\frac{\pi}{2}}$ $I = \frac{\pi}{2} \left[\left(\frac{\pi}{2} - \frac{\sin \pi}{2} \right) - (0 - 0) \right]$ $I = \frac{\pi}{2} \left[\frac{\pi}{2} - 0 \right]$ $I = \frac{\pi^2}{4} \quad \text{ans.}$
Q.3)	$\int_1^2 \frac{\sqrt{x}}{\sqrt{3-x} + \sqrt{x}} dx$
Sol.3)	$I = \int_1^2 \frac{\sqrt{x}}{\sqrt{3-x} + \sqrt{x}} dx \quad \dots\dots(1)$

	$I = \int_1^2 \frac{\sqrt{1+2-x}}{\sqrt{3-(1+2-x)} + \sqrt{(1+2-x)}} dx \quad \dots \dots \dots \left[\int_a^b f(x) dx = \int_a^b f(a+b-x) dx \right]$ $I = \int_1^2 \frac{\sqrt{3-x}}{\sqrt{x} + \sqrt{3-x}} dx \quad \dots \dots \dots (2)$ $(1) + (2)$ $2I = \int_1^2 \frac{\sqrt{x} + \sqrt{3-x}}{\sqrt{3-x} + \sqrt{x}} dx$ $2I = \int_1^2 1 \cdot dx$ $2I = (x)_1^2$ $2I = 2 - 1$ $\Rightarrow I = \frac{1}{2} \quad \text{ans.}$
Q.4)	$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1 + \sqrt{\cot x}} dx$
Sol.4)	$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1 + \sqrt{\cot x}} dx \quad \dots \dots \dots (1)$ $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1 + \sqrt{\cot(\frac{\pi}{6} + \frac{\pi}{3} - x)}} dx \quad \dots \dots \dots \text{(P-V) (above)}$ $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1 + \sqrt{\tan x}} dx$ $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1 + \frac{1}{\sqrt{\cot x}}} dx$ $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cot x}}{\sqrt{\cot x} + 1} dx \quad \dots \dots \dots (2)$ $(1) + (2)$ $2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cot x} + 1}{\sqrt{\cot x} + 1} dx$ $2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 1 \cdot dx$ $2I = (x)_{\frac{\pi}{6}}^{\frac{\pi}{3}}$ $2I = \frac{\pi}{3} - \frac{\pi}{6}$ $2I = \frac{\pi}{6}$ $I = \frac{\pi}{12} \quad \text{ans.}$
Q.5)	$I = \int_0^1 5x - 3 _{(3/5)} dx$
Sol.5)	$I = \int_0^1 5x - 3 _{(3/5)} dx$

	$I = -\int_0^{\frac{3}{5}}(5x - 3) dx + \int_{\frac{3}{5}}^1(5x - 3) dx$ $I = -\left[\frac{5x^2}{2} - 3x\right]_0^{\frac{3}{5}} + \left[\frac{5x^2}{2} - 3x\right]_{\frac{3}{5}}^1$ $I = -\left[\frac{5}{2} \cdot \frac{9}{25} - 3 \cdot \frac{3}{5}\right] + \left[\left(\frac{5}{2} - 3\right) - \left(\frac{5}{2} \cdot \frac{9}{25} - 3 \cdot \frac{3}{5}\right)\right]$ $I = -\left(\frac{9}{10} - \frac{9}{5}\right) + \left[-\frac{1}{2} - \frac{9}{10} + \frac{9}{5}\right]$ $I = -\frac{9}{10} + \frac{9}{5} - \frac{1}{2} - \frac{9}{10} + \frac{9}{5}$ $I = \frac{13}{10} \quad \text{ans.}$
Q.6)	$I = \int_{-5}^5 \frac{ x-2 }{(2)} dx$
Sol.6)	$I = \int_{-5}^5 \frac{ x-2 }{(2)} dx$ $I = -\int_5^2(x-2) dx + \int_2^5(x-2) dx$ $I = -\left[\frac{x^2}{2} - 2x\right]_{-5}^2 + \left[\frac{x^2}{2} - 2x\right]_2^5$ $I = -\left[(2-4) - \left(\frac{25}{2} + 10\right)\right] + \left[\left(\frac{25}{2} - 10\right) - (2-4)\right]$ $I = -\left[-2 - \frac{45}{2}\right] + \left[\frac{5}{2} + 2\right]$ $I = 2 + \frac{45}{2} + \frac{5}{2} + 2 = 29 \quad \text{ans.}$
Q.7)	$I = \int_1^5 \frac{ x-6 }{(6)} dx$
Sol.7)	$I = \int_1^5 \frac{ x-6 }{(6)} dx$ $I = -\int_1^5(x-6) dx$ $I = -\left[\frac{x^2}{2} - 6x\right]_1^5$ $I = -\left[\left(\frac{25}{2} - 30\right) - \left(\frac{1}{2} - 6\right)\right]$ $I = -\left[\frac{-35}{2} + \frac{11}{2}\right] = 12 \quad \text{ans.}$
Q.8)	$I = \int_{-1}^1 \frac{e^{ x }}{(0)} dx$
Sol.8)	$I = \int_{-1}^0 e^{-x} dx + \int_0^1 e^x dx$ $I = \left[\frac{e^{-x}}{-1}\right]_{-1}^0 + [e^x]_0^1$

	$I = \left[\frac{1}{-1} - \frac{e^1}{-1} \right] + [e^1 - e^0]$ $I = [-1 + e] + [e - 1]$ $I = 2e - 2 \quad \text{ans.}$
Q.9)	$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin x + \cos x dx$
Sol.9)	$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin x + \cos x dx$ $I = \int_{-\frac{\pi}{2}}^0 \sin(-x) + \cos(-x) dx + \int_0^{\frac{\pi}{2}} \sin(x) + \cos(x) dx$ $I = \int_{-\pi/2}^0 -\sin x + \cos x dx + \int_0^{\pi/2} \sin x + \cos x dx$ $I = [\cos x + \sin x]_{-\frac{\pi}{2}}^0 + [-\cos x + \sin x]_{\frac{\pi}{2}}^0$ $I = [\cos 0 + \sin 0] - \left(\cos\left(-\frac{\pi}{2}\right) + \sin\left(-\frac{\pi}{2}\right) \right) + \left[(-\cos \frac{\pi}{2} + \sin \frac{\pi}{2}) - (-\cos 0 + \sin 0) \right]$ $I = [(1 + 0) - (0 - 1)] + [(0 + 1) - (-1 + 0)]$ $I = 2 + 2 = 4 \quad \text{ans.}$
Q.10)	$I = \int_0^2 x^2 + 2x - 3 dx$
Sol.10)	$I = \int_0^2 \begin{matrix} (x+3)(x-1) \\ (-3) \quad (1) \end{matrix} dx$ $\begin{array}{c} - + \\ 0 \quad 1 \quad 2 \end{array}$ $\therefore I = -\int_0^1 (x^2 + 2x - 3) dx + \int_1^2 (x^2 + 2x - 3) dx$ $I = -\left[\frac{x^3}{3} + x^2 - 3x \right]_0^1 + \left[\frac{x^3}{3} + x^2 - 3x \right]_1^2$ $I = -\left[\left(\frac{1}{3} + 1 - 3 \right) - (0) \right] + \left[\left(\frac{8}{3} + 4 - 6 \right) - \left(\frac{1}{3} + 1 - 3 \right) \right]$ $I = -\left[\frac{-5}{3} \right] + \left[\frac{2}{3} + \frac{5}{3} \right]$ $I = \frac{12}{3} = 4 \quad \text{ans.}$