# SOLUTIONS

1. Let $I = \int \frac{10x^9 + 10^x \log_e 10}{10^x + x^{10}} dx$

Put $10^x + x^{10} = t$

$\Rightarrow (10^x \log_e 10 + 10x^9)dx = dt$

$\therefore \quad I = \int \frac{10x^9 + 10^x \log_e 10}{10^x + x^{10}} dx = \int \frac{dt}{t}$

$= \log_e t + C = \log_e(10^x + x^{10}) + C$

2. We have, $y = be^x + ce^{2x}$ ...(i)

Differentiating (i) with respect to $x$, we get

$\frac{dy}{dx} = be^x + 2ce^{2x}$ ...(ii)

Again differentiating (ii) with respect to $x$, we get

$\frac{d^2y}{dx^2} = be^x + 4ce^{2x}$

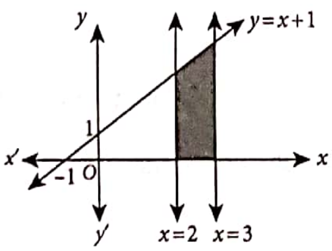
$\therefore \quad \frac{d^2y}{dx^2} - \frac{3dy}{dx} + 2y$

$= be^x + 4ce^{2x} - 3(be^x + 2ce^{2x}) + 2(be^x + ce^{2x})$

$= be^x + 4ce^{2x} - 3be^x - 6ce^{2x} + 2be^x + 2ce^{2x} = 0$

So, $y = be^x + ce^{2x}$ satisfies the given differential equation. Hence, it is a solution of the given differential equation.

3. We have, $y = x + 1$, which is a straight line




$\therefore$ Required area

$= \int_2^3 (x+1)\,dx = \left[\frac{x^2}{2} + x\right]_2^3 = \left(\frac{9}{2} + 3\right) - \left(\frac{4}{2} + 2\right)$

$= \frac{15}{2} - 4 = \frac{7}{2}$ sq. units

4. Let $I = \int (2\tan x - 3\cot x)^2 dx$

$\Rightarrow \quad I = \int (4\tan^2 x + 9\cot^2 x - 12\tan x\, \cot x)dx$

$\Rightarrow \quad I = \int \{4(\sec^2 x - 1) + 9(\text{cosec}^2 x - 1) - 12\}\, dx$

$\Rightarrow \quad I = \int (4\sec^2 x + 9\,\text{cosec}^2 x - 25)dx$

$\Rightarrow \quad I = 4\tan x - 9\cot x - 25x + C$

5. We have, $5x\left(\frac{dy}{dx}\right)^2 - \frac{d^2y}{dx^2} - 6y = \log x.$

Highest order derivative is $\frac{d^2y}{dx^2}$, so order is 2.

Now, given differential equation is polynomial in differential coefficients and power of $\frac{d^2y}{dx^2}$ is one, so degree is 1.

**OR**

The given differential equation is

$\frac{dy}{dx} + y\tan x - \sec x = 0$

It is a linear differential equation.

$\therefore \quad$ I.F. $= e^{\int \tan x\, dx} = e^{\log \sec x} = \sec x$

6. Let $A, B, C$ be the respective events of solving the problem. Then, $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ and $P(C) = \frac{1}{4}$.

Clearly $A, B, C$ are independent events and the problem is solved if at least one student solves it.

$\therefore \quad$ Required probability $= P(A \cup B \cup C)$

$= 1 - P(\bar{A})P(\bar{B})P(\bar{C})$

$= 1 - \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right) = 1 - \frac{1}{4} = \frac{3}{4}$

7. Let $A, B, C, D$ denote events of getting a white ball in first, second, third and fourth draw respectively.

Required probability $= P(A \cap B \cap C \cap D)$

$= P(A)P(B \mid A)P(C \mid A \cap B)P(D \mid A \cap B \cap C)$ ...(i)

Now, $P(A) = \frac{5}{20} = \frac{1}{4}$.

When a white ball is drawn in the first draw, there are 19 balls left in the bag out of which 4 are white.

$\therefore \quad P(B \mid A) = \frac{4}{19}$

Since the ball drawn is not replaced, therefore after drawing a white ball in second draw there are 18 balls left in the bag out which 3 are white.

$\therefore \quad P(C \mid A \cap B) = \frac{3}{18} = \frac{1}{6}$

After drawing a white ball in third draw, there are 17 balls left in the bag, out of which 2 are white.

$\therefore\ P(D \mid A \cap B \cap C) = \frac{2}{17}$

$\therefore$ Required probability $= \frac{1}{4} \times \frac{4}{19} \times \frac{1}{6} \times \frac{2}{17} = \frac{1}{969}$

**8.** We have, $\vec{r} = \hat{i} + \hat{j} + \lambda(2\hat{i} - \hat{j} + \hat{k})$ ...(i)

and $\vec{r} = 2\hat{i} + \hat{j} - \hat{k} + \mu(3\hat{i} - 5\hat{j} + 2\hat{k})$ ...(ii)

Here, $\vec{a}_1 = \hat{i} + \hat{j}, \vec{b}_1 = 2\hat{i} - \hat{j} + \hat{k}$,

$\vec{a}_2 = 2\hat{i} + \hat{j} - \hat{k}, \vec{b}_2 = 3\hat{i} - 5\hat{j} + 2\hat{k}$

Therefore, $\vec{a}_2 - \vec{a}_1 = (2\hat{i} + \hat{j} - \hat{k}) - (\hat{i} + \hat{j}) = \hat{i} - \hat{k}$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix} = (-2+5)\hat{i} - (4-3)\hat{j} + (-10+3)\hat{k}$$

$\Rightarrow \vec{b}_1 \times \vec{b}_2 = 3\hat{i} - \hat{j} - 7\hat{k}$

So, $|\vec{b}_1 \times \vec{b}_2| = \sqrt{(3)^2 + (-1)^2 + (-7)^2} = \sqrt{9 + 1 + 49}$

$= \sqrt{59}$

Hence, the shortest distance between two given lines be

$$d = \left| \frac{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|} \right| = \left| \frac{(\hat{i} - \hat{k}) \cdot (3\hat{i} - \hat{j} - 7\hat{k})}{\sqrt{59}} \right|$$

$\Rightarrow d = \frac{3 - 0 + 7}{\sqrt{59}} = \frac{10}{\sqrt{59}}$ units

**OR**

The equation of the plane passing through the line of intersection of the planes $2x + y - z = 3$ and $5x - 3y + 4z + 9 = 0$ is

$(2x + y - z - 3) + \lambda(5x - 3y + 4z + 9) = 0$

$\Rightarrow x(2 + 5\lambda) + y(1 - 3\lambda) + z(4\lambda - 1) + 9\lambda - 3 = 0$ ...(i)

The plane (i) is parallel to the line

$\frac{x-1}{2} = \frac{y-3}{4} = \frac{z-5}{5}$

$\therefore\ 2(2 + 5\lambda) + 4(1 - 3\lambda) + 5(4\lambda - 1) = 0$

$\Rightarrow 18\lambda + 3 = 0 \Rightarrow \lambda = -\frac{1}{6}$

Putting the value of $\lambda$ in (i), we obtain

$x\left(2 - \frac{5}{6}\right) + y\left(1 + \frac{3}{6}\right) + z\left(-\frac{4}{6} - 1\right) - \frac{9}{6} - 3 = 0$

$\Rightarrow 7x + 9y - 10z - 27 = 0$, which is the equation of the required plane.

**9.** We have, $P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B})$

$\Rightarrow P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B)$

$\Rightarrow P(\bar{A} \cap \bar{B}) = 1 - \{P(A) + P(B) - P(A \cap B)\}$

$\Rightarrow P(\bar{A} \cap \bar{B}) = 1 - \left\{\frac{3}{8} + \frac{1}{2} - \frac{1}{4}\right\} = \frac{3}{8}$

Now, $P(\bar{A}) = 1 - P(A) = \frac{5}{8}$ and $P(\bar{B}) = 1 - P(B) = \frac{1}{2}$

$$\therefore\ P(\bar{A} \mid \bar{B}) = \frac{P(\bar{A} \cap \bar{B})}{P(\bar{B})} = \frac{\frac{3}{8}}{\frac{1}{2}} = \frac{3}{4}$$

$$\text{and } P(\bar{B} \mid \bar{A}) = \frac{P(\bar{A} \cap \bar{B})}{P(\bar{A})} = \frac{\frac{3}{8}}{\frac{5}{8}} = \frac{3}{5}$$

**OR**

We have, $A = \{(1, 1)\}, B = \{(1, 2), (2, 1)\}$ and $C = \{(1, 3), (3, 1), (2, 2)\}$

(i) Since $A$ consists of a single sample point, it is a simple event.

(ii) Since both $B$ and $C$ contain more than one sample point, therefore each one of them is a compound event.

(iii) Since $A \cap B = \phi$.

$\therefore$ $A$ and $B$ are mutually exclusive events.

**10.** Let $\theta$ be the angle between vectors $\vec{a}$ and $\vec{b}$.

We have, $|\vec{a}| = |\vec{b}| = \sqrt{2}$ and $\vec{a} \cdot \vec{b} = -1$

$\therefore\ \cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$

$\Rightarrow \cos\theta = \frac{-1}{\sqrt{2} \times \sqrt{2}} = -\frac{1}{2}$

$\Rightarrow \cos\theta = \cos\frac{2\pi}{3}$ $[\because\ 0 \le \theta \le \pi]$

$\Rightarrow \theta = \frac{2\pi}{3}$

Hence, the angle between $\vec{a}$ and $\vec{b}$ is $\frac{2\pi}{3}$.

**11.** (i) If vectors $\vec{a}$ and $\vec{b}$ are perpendicular, then $\vec{a} \cdot \vec{b} = 0$

$\Rightarrow (3\hat{i} + 2\hat{j} + 9\hat{k}) \cdot (\hat{i} + p\hat{j} + 3\hat{k}) = 0$

$\Rightarrow 3 + 2p + 27 = 0 \Rightarrow p = -15$

(ii) We know that, the vectors $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ are parallel iff $\vec{a} = \lambda\vec{b}$

$\Leftrightarrow (a_1\hat{i} + a_2\hat{j} + a_3\hat{k}) = \lambda(b_1\hat{i} + b_2\hat{j} + b_3\hat{k})$

$\Leftrightarrow a_1 = \lambda b_1, a_2 = \lambda b_2, a_3 = \lambda b_3$

$\Leftrightarrow \frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3} = \lambda$

So, given vectors $\vec{a} = 3\hat{i} + 2\hat{j} + 9\hat{k}$ and $\vec{b} = \hat{i} + p\hat{j} + 3\hat{k}$ will be parallel iff

$$\frac{3}{1}=\frac{2}{p}=\frac{9}{3} \Rightarrow 3=\frac{2}{p} \Rightarrow p=\frac{2}{3}$$

**OR**

$\vec{a}\times\vec{b}$ is defined as $\vec{a}\times\vec{b}=|\vec{a}||\vec{b}|\sin\theta\,\hat{n}$ where, $\theta$ is the angle between $\vec{a}$ and $\vec{b}$, $0\le\theta\le\pi$ and $\hat{n}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$, such that $\vec{a}$, $\vec{b}$ and $\hat{n}$ form a right handed system.

We know that, $|\vec{a}\times\vec{b}|=|\vec{a}||\vec{b}|\sin\theta$ ...(i)

and $|\vec{a}\cdot\vec{b}|=|\vec{a}||\vec{b}|\cos\theta$ ... (ii)

Dividing (i) by (ii), we get

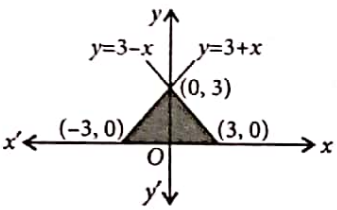
$$\frac{|\vec{a}\times\vec{b}|}{|\vec{a}\cdot\vec{b}|}=\frac{|\vec{a}||\vec{b}|\sin\theta}{|\vec{a}||\vec{b}|\cos\theta} \Rightarrow |\vec{a}\times\vec{b}|=|\vec{a}\cdot\vec{b}|\tan\theta$$

**12.** We have, $y=3-|x|$

$\Rightarrow y=3+x,\ \forall\ x<0$ ...(i)

and $y=3-x,\ \forall\ x\ge 0$ ...(ii)

$\therefore$ Required area = area of shaded region




$$=\left|2\int_{-3}^{0}(3+x)\,dx\right|=\left|2\left[3x+\frac{x^2}{2}\right]_{-3}^{0}\right|$$

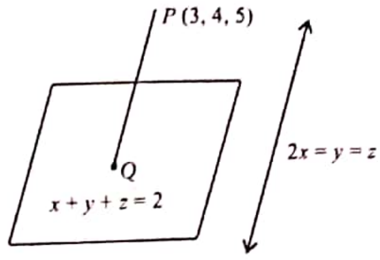
$$=\left|-2\left[-9+\frac{9}{2}\right]\right|$$

$$=\left|-2\times\frac{-9}{2}\right|=9 \text{ sq. units}$$

**13.** We have equation of line as $2x=y=z$

or $\frac{x}{1}=\frac{y}{2}=\frac{z}{2}$

Now, equation of line through the point $P(3, 4, 5)$ and parallel to the given line is




$$\frac{x-3}{1}=\frac{y-4}{2}=\frac{z-5}{2}=\lambda$$

Coordinates of any point $Q$ on this line are $(\lambda+3, 2\lambda+4, 2\lambda+5)$

Since this point must lie on the plane $x+y+z=2$

$\therefore\ \lambda+3+2\lambda+4+2\lambda+5=2$

$\Rightarrow 5\lambda+12=2 \Rightarrow \lambda=-2$

$\therefore$ The coordinates of the point $Q$ are $(1, 0, 1)$.

$$\therefore\ PQ=\sqrt{(3-1)^2+(4-0)^2+(5-1)^2}$$

$$=\sqrt{4+16+16}=6$$

Hence, required distance of the point from the plane is 6 units.

**14.** (i) The given differential equation is

$$(1+\sin x)\frac{dy}{dx}+y\cos x+x=0.$$

$$\Rightarrow \frac{dy}{dx}+\frac{y\cos x}{1+\sin x}=\frac{-x}{1+\sin x}$$

$$\text{I.F.}=e^{\int P\,dx}=e^{\int\frac{\cos x}{1+\sin x}dx}$$

Put $1+\sin x=t \Rightarrow \cos x\,dx=dt$

$$\therefore\ \text{I.F.}=e^{\int\frac{1}{t}dt}=e^{\log t}=t=1+\sin x$$

(ii) Solution of given differential equation is given by

$$y.(\text{I.F.})=\int Q\,(\text{I.F.})\,dx+c$$

$$\Rightarrow y\,(1+\sin x)=\int\frac{-x}{1+\sin x}.(1+\sin x)\,dx+c$$

$$\Rightarrow y\,(1+\sin x)=\frac{-x^2}{2}+c$$

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