DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth **SUBJECT : MATHS**
DATE : **DPP NO. : 1**

Topic :- CONTINUITY AND DIFFERENTIABILITY

1. Let $[x]$ denotes the greatest integer less than or equal to x and $f(x) = [\tan^2 x]$. Then,
   a) $\lim_{x\to 0} f(x)$ does not exist
   b) $f(x)$ is continuous at $x = 0$
   c) $f(x)$ is not differentiable at $x = 0$
   d) $f'(0) = 1$

2. The value of $f(0)$ so that $\frac{(-e^x + 2^x)}{x}$ may be continuous at $x = 0$ is
   a) $\log\left(\frac{1}{2}\right)$ b) 0 c) 4 d) $-1 + \log 2$

3. Let $f(x)$ be an even function. Then $f'(x)$
   a) Is an even function b) Is an odd function c) May be even or odd d) None of these

4. If $f(x) = \begin{cases} [\cos \pi\ x], x < 1 \\ |x-2|,\ 2 > x \geq 1 \end{cases}$, then $f(x)$ is
   a) Discontinuous and non-differentiable at $x = -1$ and $x = 1$
   b) Continuous and differentiable at $x = 0$
   c) Discontinuous at $x = 1/2$
   d) Continuous but not differentiable at $x = 2$

5. If $f(x) = \begin{cases} \frac{|x+2|}{\tan^{-1}(x+2)}, x \neq -2 \\ 2, \quad x = -2 \end{cases}$, then $f(x)$ is
   a) Continuous at $x = -2$
   b) Not continuous $x = -2$
   c) Differentiable at $x = -2$
   d) Continuous but not derivable at $x = -2$

6. If $f(x) = |\log|x||$, then
   a) $f(x)$ is continuous and differentiable for all x in its domain
   b) $f(x)$ is continuous for all x in its domain but not differentiable at $x = \pm 1$
   c) $f(x)$ is neither continuous nor differentiable at $x = \pm 1$
   d) None of the above

7. If $f'(a) = 2$ and $f(a) = 4$, then $\lim_{x\to a} \frac{xf(a) - af(x)}{x - a}$ equals
   a) $2a - 4$ b) $4 - 2a$ c) $2a + 4$ d) None of these

8. If $f(x) = x(\sqrt{x} + \sqrt{x+1})$, then
   a) $f(x)$ is continuous but not differentiable at $x = 0$ b) $f(x)$ is differentiable at $x = 0$
   c) $f(x)$ is not differentiable at $x = 0$ d) None of the above

9. If $f(x) = \begin{cases} ax^2 + b, & b \neq 0, x \leq 1 \\ x^2 b + ax + c, & x > 1 \end{cases}$, then, $f(x)$ is continuous and differentiable at $x = 1$, if
   a) $c = 0, a = 2b$ b) $a = b, c \in R$ c) $a = b\,, c = 0$ d) $a = b, c \neq 0$

10. For the function $f(x) = \begin{cases} |x-3|, & x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, & x < 1 \end{cases}$ which one of the following is incorrect?
   a) Continuous at $x = 1$ b) Derivable at $x = 1$ c) Continuous at $x = 3$ d) Derivable at $x = 3$

11. If f:$R \to R$ is defined by
$$f(x) = \begin{cases} \dfrac{2\sin x - \sin 2x}{2x\cos x}, & \text{if } x \neq 0, \\ a, & \text{if } x = 0 \end{cases}$$
Then the value of a so that f is continuous at 0 is
   a) 2 b) 1 c) -1 d) 0

12. $f(x) = x + |x|$ is continuous for
   a) $x \in (-\infty, \infty)$ b) $x \in (-\infty, \infty) - \{0\}$ c) Only $x > 0$ d) No value of x

13. If the function
$$f(x) = \begin{cases} \{1 + |\sin x|\}^{\frac{a}{|\sin x|}}, & -\dfrac{\pi}{6} < x < 0 \\ b, & x = 0 \\ e^{\frac{\tan 2x}{\tan 3x}}, & 0 < x < \dfrac{\pi}{6} \end{cases}$$
Is continuous at $x = 0$
   a) $a = \log_e b, b = \frac{2}{3}$ b) $b = \log_e a, a = \frac{2}{3}$ c) $a = \log_e b, b = 2$ d) None of these

14. If $f(x) = x^2 + \frac{x^2}{1+x^2} + \frac{x^2}{(1+x^2)^2} + \ldots + \frac{x^2}{(1+x^2)^n} + \ldots$, then at $x = 0$, $f(x)$
   a) Has no limit
   b) Is discontinuous
   c) Is continuous but not differentiable
   d) Is differentiable

15. Let$f(x) = \begin{cases} 1, & \forall \quad x < 0 \\ 1 + \sin x, & \forall \quad 0 \leq x \leq \pi/2 \end{cases}$, then what is the value of $f'(x)$ at $x = 0$?
   a) 1 b) -1 c) ∞ d) Does not exist

16. The function $f(x) = x - |x - x^2|$ is
   a) Continuous at $x = 1$ b) Discontinuous at $x = 1$
   c) Not defined at $x = 1$ d) None of the above

17. If $f(x+y+z)=f(x).f(y).f(z)$ for all x,y,z and $f(2)=4$, $f'(0)=3$, then $f'(2)$ equals

a) 12 b) 9 c) 16 d) 6

18. If $f(x)=|\log_e|x||$, then $f'(x)$ equals

a) $\frac{1}{|x|}$, $x\neq 0$

b) $\frac{1}{x}$ for $|x|>1$ and $\frac{-1}{x}$ for $|x|<1$

c) $\frac{-1}{x}$ for $|x|>1$ and $\frac{1}{x}$ for $|x|<1$

d) $\frac{1}{x}$ for $|x|>0$ and $-\frac{1}{x}$ for $x<0$

19. If the function $f(x)=\begin{cases}\frac{1-\cos x}{x^2}, & \text{for } x\neq 0\\ k, & \text{for } x=0\end{cases}$ is continuous at $x=0$, then the value of k is

a) 1 b) 0 c) $\frac{1}{2}$ d) -1

20. Function $f(x)=|x-1|+|x-2|$, $x\in R$ is

a) Differentiable everywhere in R

b) Except $x=1$ and $x=2$ differentiable everywhere in R

c) Not continuous at $x=1$ and $x=2$ d) Increasing in R