UNIT 1 ELECTROSTATICS

EASY & SCORINGAREAS (MLL)

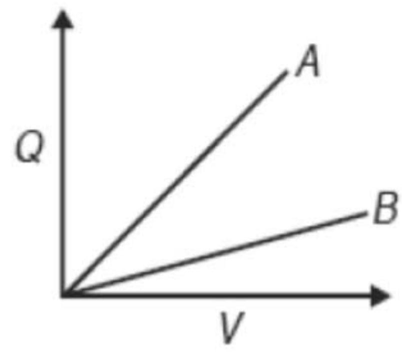
Coulombs law
Electric dipole-electric field on axial and equatorial line, torque acting on the dipole.
Statement of Gauss Theorem.
Electric field due to infinite plane sheet of charge (Application of Gauss Theorem)
Electric field due to spherical shell (Application of Gauss Theorem)
Electric field due to infinite uniformly charged line charge (Application of Gauss Theorem)
Electric potential due to dipole and point charge.
Electrostatic Potential energy and equipotential surfaces
Electric lines of force and its properties
Capacity of a parallel plate capacitor with (i) air (ii) dielectric (iii) conducting medium between the plates
Numericals on series and parallel combination of capacitor.
Energy stored in a capacitor.

ONE MARK QUESTIONS

1. Define dipole moment of an electric dipole. Is it a scalar or a vector?
2. In which orientation a dipole placed in a uniform electric field is in a) Stable, b) Unstable Equilibrium?
3. What is the electric potential due to electric dipole at an equatorial point?
4. What is the shape of equipotential surface due to a single isolated charge?
5. Name a physical quantity whose SI unit is J/C. Is it a scalar or a vector quantity?
6. A hollow metal sphere of radius 5 cm is charged such that the potential on its surface is 10V. What is the potential at the centre of the sphere?

TWO MARKS QUESTION

1. What is the work done to move a test charge q through a distance of 1 cm along the equatorial axis of dipole?
2. **A** 500μC charge is at the centre of square of side 10cm. Find work done in moving a charge of 10 μC between two diagonally opposite points on the square.
3. Can two equipotential surfaces intersect each other? Give reasons.
4. The given graph shows the variation of charge, q versus potential difference V for capacitors C1 and C2 . The two capacitors have same plate area of C2 is double than that C1. Which of the lines in the graph correspond to C1 and C2 and why?



5. Depict the equipotential surfaces for a system of two identical positive point charges placed at a distance ‘d’ apart.

(3 MARKS & 5 MARKS QUESTIONS)

1. Derive expression for electric field at a point on the axial line of the dipole. Give the direction of electric field at the point.
2. Derive expression for electric field at a point on the equatorial line of dipole.
3. An electric dipole is held in uniform electric field
   (i) Show that no net force acts on it.
   (ii) Derive an expression for the torque acting on it
4. State Gauss Theorem. A thin charged wire of infinite length has line charge density ‘λ’. Derive expression for electric field at a distance ‘r’.
5. Charge q is distributed uniformly on a spherical shell of radius R. Using gauss law derive expression of electric field at a distance r from the centre when (i)$r>R$ (ii) $r=R$ (iii) $r<R$

6. Derive expression for capacitance of parallel plate capacitor.

7. Derive expression for capacitance of parallel plate capacitor with dielectric as medium between the plates.

8. Derive expression for energy stored in a capacitor.

VALUE BASED QUESTIONS (ELECTROSTATICS)

1. Mr. Bose was driving on a highway along fields. When a drizzle starts with lightning and thunder storm. He spots a few farmers walking with iron spoke top umbrellas to avoid getting wet. He stops his car and instructs his co passengers to keep sitting inside the car. He advises the farmers not to use the umbrella till the lightning subsides.

a. What are the two human qualities which Mr. Bose exhibited?
Ans.: caring attitude, scientific temper, presence of mind.
b. Why did he advise the farmers not to use the type of umbrella they were using?
c. Why did he advise his co passengers to set inside the car and not to venture out?

ANSWERS

One mark question solution

Ans 1 Electric dipole moment of an electric dipole is equal to the product of either charge or distance between the two charges.

$$p = q \times 2a$$

Where p is dipole moment.
It is a scalar quantity.

Ans 2. (a) For stable equilibrium the angle between p and E must be 00
(b) For unstable equilibrium the angle between p and E must be 1800

Ans 3. **Potential at a point on equatorial line is 0.**

Ans 4. For an isolated charge equipotential surface are concentric spherical shells and distance between them increases with the decrease in field.

Ans 5. J/C is unit of electric potential. It is a scalar quantity.
Ans6. 10V

2 marks question solution

Ans 1. Potential at any point on the equatorial line is 0. Hence work done W = qΔV =0 as ΔV=0
Ans2. Two diagonally opposite points are equidistant from the centre of square hence potential at these points due to given charge will be equal.

W= QΔV=0 as ΔV=0.
Ans 3. No, two equipotential surfaces cannot intersect each other because two normals can be drawn at intersecting point on the two surfaces which gives two directions of E at the same point which is not possible.

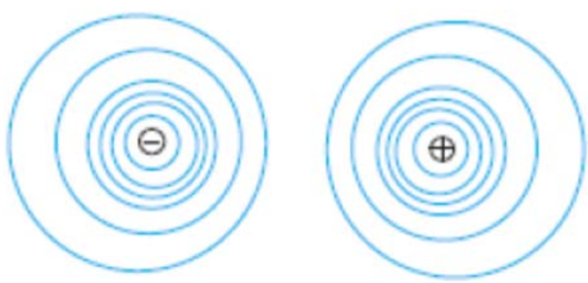
Ans 4.

$$C = \frac{\epsilon_0 A}{d}$$

$$C_2 = 2C_1$$

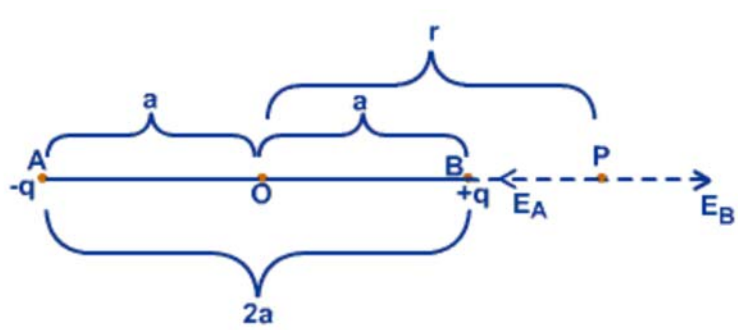
The slope of graph represents capacity of capacitor
A has greater slope than that of B
So capacitance of A is greater than that of B

Ans 5 Equipotential surfaces for two equal and opposite charges



Long question solution

Ans 1. Electric field at a point on an axial point of electric dipole.



The axial line of a dipole is the line passing through the positive and negative charges of the electric dipole.

Consider a system of charges (-q and +q) separated by a distance 2a. Let 'P' be any point on an axis where the field intensity is to be determined.
Electric field at P (EB) due to +q

$$E_B = \frac{1}{4\pi\varepsilon_0}\frac{q}{(BP)^2} \quad \text{along BP}$$
$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-a)^2}$$

Electric field at P due to -q (EA)

$$E_A = \frac{1}{4\pi\varepsilon_0}\frac{q}{(AP)^2} \quad \text{along PA}$$
$$= \frac{1}{4\pi\varepsilon_0}\frac{1}{(r+a)^2}$$

Net field at P is given by

$$E_p = E_B - E_A$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{(r-a)^2} - \frac{q}{(r+a)^2}\right]$$

Simplifying, we get

$$E_p = \frac{q}{4\pi\varepsilon_0}\frac{4ar}{\left(r^2-a^2\right)^2}$$

$$E_p = \frac{2qa}{4\pi\varepsilon_0}\frac{2r}{\left(r^2-a^2\right)^2}$$

$$\left(2aq=p\ ,\ K = \frac{1}{4\pi\varepsilon_0}\right)$$

$$\text{or } E_p = \frac{2kpr}{\left(r^2-a^2\right)^2} \quad \text{along BP}$$

As a special case :

$$\text{If } 2a \ll r,\ E_p = \frac{2kp}{r^3} \quad \text{along BP}$$

Ans 2. An equatorial line of a dipole is the line perpendicular to the axial line and passing through a point mid way between the charges.

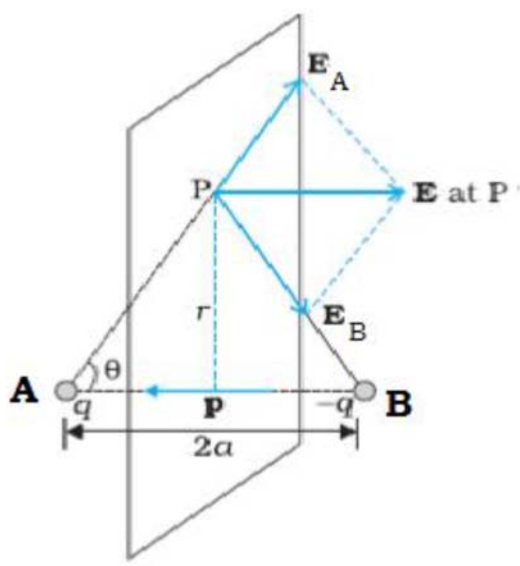
Consider a dipole consisting of -q and +q separated by a distance 2a. Let P be a point Consider a point P on the equatorial line.

$$\vec{E_A} = \frac{1}{4\pi\varepsilon_0}\frac{q}{(AP)^2} \text{ along PA}$$

$$\vec{E_A} = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r^2 + a^2)}$$

$$\vec{E_B} = \frac{1}{4\pi\varepsilon_0}\frac{q}{(BP)^2} \text{ along BP}$$

$$\vec{E_B} = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r^2 + a^2)}$$



The resultant intensity is the vector sum of the intensities along PA and PB. EA and EB can be resolved into vertical and horizontal components. The vertical components of EA Sinθ and EB Sinθ cancel each other as they are equal and oppositely directed. It is the horizontal components which add up to give the resultant field.

$$E = E_A \cos\theta + E_B \cos\theta$$

$$E_A = E_B = \frac{1}{4\pi\varepsilon_0}\frac{q}{\left(\sqrt{r^2 + a^2}\right)^2} = \frac{1}{4\pi\varepsilon_0}\frac{q}{\left(r^2 + a^2\right)}$$

E = 2EA cos �

Substituting, $\cos\theta = \dfrac{a}{\left(r^2 + a^2\right)^{\frac{1}{2}}}$ in the above equation

$$E = 2E_A \cos\theta = \frac{2}{4\pi\varepsilon_0}\frac{q}{\left(r^2 + a^2\right)}\frac{a}{\left(r^2 + a^2\right)^{\frac{1}{2}}}$$

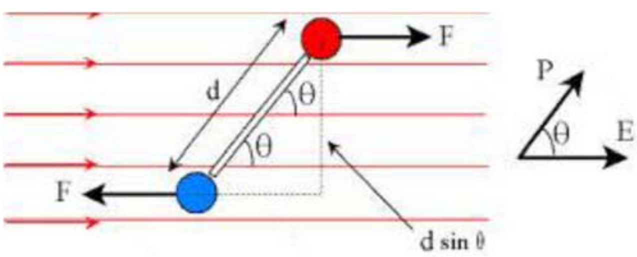
$$E = \frac{kp}{\left(r^2 + a^2\right)^{\frac{3}{2}}} \quad \text{along } P_x$$

As 2qa = p

As a special case,

If $2a \ll r$ then, $E = \dfrac{kp}{r^3}$ along P_x

Ans 3.



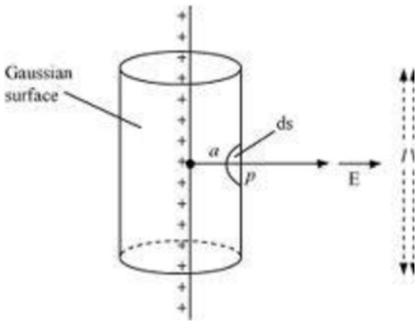
Force on +q charge=qE along direction of E
Force on –q charge =qE opposite to E
Fnet=qE-qE =0
The forces are equal in magnitude, opposite in direction acting at different points, therefore they form a couple which rotates the dipole.

$$\text{Torque}\tau = F \times perp.distance$$

$$\tau = F \times d\sin\theta = qE \times d\sin\theta = (qd)E\sin\theta$$

$$\left[\tau = pE\sin\theta \;Or\; \vec{\tau} = \vec{p} \times \vec{E}\right]$$

Ans 4. Gauss's Law: 'Electric flux over a closed surface is 1/ε0 times the charge enclosed by it.'



$$\emptyset = \frac{q}{\epsilon_0}$$

To calculate the field at P we consider a Gaussian surface with wire as axis, radius r and length l as shown in the figure.
The electric lines of force are parallel to the end faces of the cylinder and hence the component of the field along the normal to the end faces is zero.
The field is radial everywhere and hence the electric flux crosses only through the curved surface of the cylinder.

If E is the electric field intensity at P, then the electric flux through the Gaussian surface is

$$\emptyset = E \times 2\pi r l$$

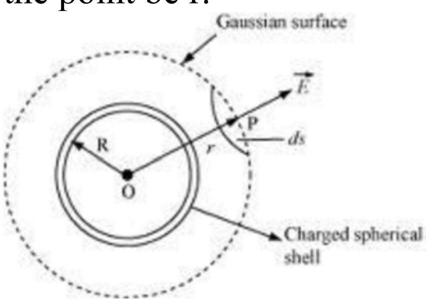
According to gauss theorem electric flux is

$$\emptyset = \frac{q}{\epsilon_0} = \frac{\lambda l}{\epsilon_0}$$

$$\text{Hence} E \times 2\pi r l = \frac{\lambda l}{\epsilon_0}$$

$$\left[\therefore \boldsymbol{E} = \frac{\boldsymbol{\lambda}}{\boldsymbol{2\pi \epsilon_0 r}}\right]$$

Ans 5. Consider a hollow conducting sphere of radius R with its centre at O. let σ be its surface density. The field at any point P, outside or inside depends upon the distance from the centre of the spherical shell. Let the distance between the centre of the spherical shell and the point be r.



Case (i) r>R

At points outside the sphere the electric field is radial every where because of spherical symmetry.

Total electric flux$\emptyset = E \times 4\pi r^2$

According to gauss theorem electric flux i

$$\emptyset = \frac{q}{\epsilon_0}$$

hence$E \times 4\pi r^2 = \frac{q}{\epsilon_0}$

$\left[E = \frac{q}{4\pi\epsilon_0 r^2}\right]$ Electric field due to charged shell is same as that due to a point charge q placed he centre of shell

Case (i) r=R

When point P lies on the surface of the shell or sphere, r = R

hence$E \times 4\pi R^2 = \frac{q}{\epsilon_0}$

$E = \frac{q}{4\pi\epsilon_0 R^2} = \frac{\sigma}{\epsilon_0}\Big]$

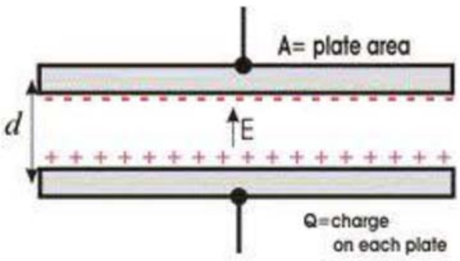
Case (i) r<R

The gaussian surface does not enclose any charge, (charge resides on the surface of the shell)

$E.4\pi R^2 = \frac{0}{\varepsilon_0}$ Hence E = 0

Ans 6. Let the surface charge density on the plates be σ such that

$$\sigma = \frac{Q}{A}$$



Electric field between the plates is given by

$$E = \frac{\sigma}{2\,\epsilon_0} + \frac{\sigma}{2\,\epsilon_0} = \frac{\sigma}{\epsilon_0}$$

Potential difference between the plates is V=Ed

$$V = \frac{\sigma}{\epsilon_0} d$$

Capacity of a capacitor $C = \frac{Q}{V} = \frac{\sigma A}{\sigma d/\epsilon_0} = \frac{\epsilon_0 A}{d}$

$$\left[C = \frac{\epsilon_0\, A}{d}\right]$$

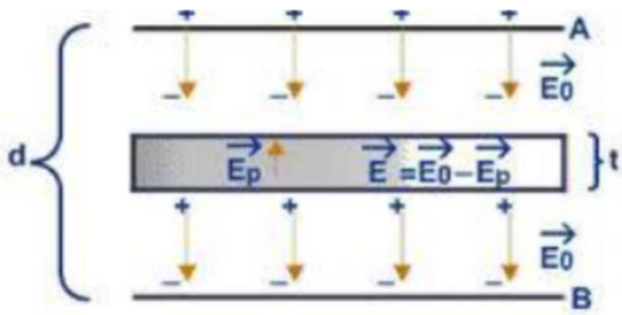
Ans 7. Let the surface charge density on the plates be σ

Such that

Such that $\sigma = \frac{Q}{A}$

Electric field between the plates is given by

$$\overrightarrow{E_0} = \frac{\sigma}{\epsilon_0} \text{ and } \overrightarrow{E_i} = \frac{\sigma}{k\epsilon_0}$$



where E0 is electric field in air and Ei is electric field in dielectric.
Potential difference between the plates is given by

$$V = \overrightarrow{E_O}(d-t) + \overrightarrow{E_i}t = \frac{\sigma}{\epsilon_0}(d-t) + \frac{\sigma}{k\,\epsilon_0}t = \frac{\sigma}{\epsilon_0}\left(d - t + \frac{t}{k}\right)$$

Capacity of a capacitor $C = \frac{Q}{V} = \frac{\sigma A}{\frac{\sigma}{\epsilon_0}\left(d-t+\frac{t}{k}\right)} = \frac{\epsilon_0 A}{\left(d-t+\frac{t}{k}\right)}$

$$\left[C = \frac{\epsilon_0 A}{d - t(1-\frac{1}{k})}\right]$$

If d= t then $\left[C = k\frac{\epsilon_0 A}{d}\right]$

Ans 8. Consider a parallel plate capacitor of capacity C. Let at any instant the charge on the capacitor be Q'. Then potential difference between the plates will be
Suppose the charge on the plates increases by d Q'. The work done will be

$$dW = V'dQ' = \frac{Q'}{C}\,dQ'$$

The total work done is $W = \int_0^Q \frac{Q'}{C}\,dQ' = \left[\frac{Q^2}{2C}\right]$

This work done is stored as electrical potential energy.

$$\left[\therefore U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}CV\right]$$